

## Symbolic vs. Algorithmic Differentiation of GSL Integration Routines

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# Symbolic vs. Algorithmic Differentiation of GSL Integration Routines

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**Abstract.** Forward and reverse modes of algorithmic differentiation (AD) transform implementations of multivariate vector functions  $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$  as computer programs into tangent and adjoint code, respectively. The adjoint mode is of particular interest in large-scale functions due to the independence of its computational cost on the number of free variables. The additional memory requirement for the computation of derivatives of the output with respect to parameters by a fully algorithmic method (derived by AD) can quickly become prohibitive for large values of  $n$ . This can be reduced significantly by the symbolic approach to differentiation of the underlying integration routine. Vectorizing gsl routines for integration and applying symbolic adjoint on them has considerably less memory requirement with nearly the same runtime overhead and in most cases faster convergence in comparison with algorithmic adjoint.

## 1 Differentiation of Integrals

Let us consider an interval which the limits of the integral are themselves functions of  $\alpha \in \mathbb{R}$ , it follows that:

$$I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(\alpha, x) dx = F(\alpha, b(\alpha)) - F(\alpha, a(\alpha)) \quad , \quad (1)$$

which yields the partial derivatives

$$\frac{\partial I}{\partial b} = f(\alpha, b(\alpha)) \quad , \quad \frac{\partial I}{\partial a} = -f(\alpha, a(\alpha)) \quad .$$

Considering chain rule and Leibniz's rule for differentiation under the integral sign [Fla73],

$$\frac{dI}{d\alpha} = f(\alpha, b(\alpha)) \frac{db}{d\alpha} - f(\alpha, a(\alpha)) \frac{da}{d\alpha} + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(\alpha, x)}{\partial \alpha} dx \quad . \quad (2)$$

Now suppose that,  $\alpha \in \mathbb{R}^n$ , i.e.,

$$I(\alpha_1, \alpha_2, \dots, \alpha_n) = \int_{a(\alpha_1, \alpha_2, \dots, \alpha_n)}^{b(\alpha_1, \alpha_2, \dots, \alpha_n)} f(\alpha_1, \alpha_2, \dots, \alpha_n, x) dx \quad . \quad (3)$$

Differentiating the above equation with respect to all parameters  $\alpha = (\alpha_1, \dots, \alpha_n)$  yields:

$$\begin{aligned}
\frac{dI}{d\alpha_1} &= f(\boldsymbol{\alpha}, b(\boldsymbol{\alpha})) \frac{db(\boldsymbol{\alpha})}{d\alpha_1} - f(\boldsymbol{\alpha}, a(\boldsymbol{\alpha})) \frac{da(\boldsymbol{\alpha})}{d\alpha_1} + \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \frac{\partial f(\boldsymbol{\alpha}, x)}{\partial \alpha_1} dx \quad , \\
\frac{dI}{d\alpha_2} &= f(\boldsymbol{\alpha}, b(\boldsymbol{\alpha})) \frac{db(\boldsymbol{\alpha})}{d\alpha_2} - f(\boldsymbol{\alpha}, a(\boldsymbol{\alpha})) \frac{da(\boldsymbol{\alpha})}{d\alpha_2} + \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \frac{\partial f(\boldsymbol{\alpha}, x)}{\partial \alpha_2} dx \quad , \\
&\dots \\
\frac{dI}{d\alpha_n} &= f(\boldsymbol{\alpha}, b(\boldsymbol{\alpha})) \frac{db(\boldsymbol{\alpha})}{d\alpha_n} - f(\boldsymbol{\alpha}, a(\boldsymbol{\alpha})) \frac{da(\boldsymbol{\alpha})}{d\alpha_n} + \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \frac{\partial f(\boldsymbol{\alpha}, x)}{\partial \alpha_n} dx \quad .
\end{aligned}$$

In other words

$$\nabla I = f(\boldsymbol{\alpha}, b(\boldsymbol{\alpha})) \nabla b - f(\boldsymbol{\alpha}, a(\boldsymbol{\alpha})) \nabla a + \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \nabla f(\boldsymbol{\alpha}, x) dx \quad . \quad (4)$$

This means that we have gradients of bounds multiplied by integrand and one quadrature instead of gradient of quadrature.

## 2 Numerical Integration in GSL

In gsl [GDT<sup>+</sup>09], there are routines for adaptive and non-adaptive integration of general functions, with specialised routines for specific cases. These include integration over infinite and semi-infinite ranges, singular integrals, including logarithmic singularities, computation of Cauchy principal values and oscillatory integrals.

Each algorithm computes an approximation to a definite integral of the form,

$$I = \int_a^b f(x)w(x)dx \quad ,$$

where  $w(x)$  is a weight function (for general integrands  $w(x) = 1$ ). The user provides absolute and relative error bounds ( $epsabs, epsrel$ ) which specify the following accuracy requirement,

$$|RESULT - I| \leq \max(epsabs, epsrel|I|) \quad ,$$

where  $RESULT$  is the numerical approximation computed by the algorithm. The algorithms attempt to estimate the absolute error  $ABSERR = |RESULT - I|$  in such a way that the following inequality holds,

$$|RESULT - I| \leq ABSERR \leq \max(epsabs, epsrel|I|) \quad .$$

In short, the routines return the first approximation which has an absolute error smaller than  $epsabs$  or a relative error smaller than  $epsrel$ .

The algorithms in QUADPACK use a naming convention based on the following letters,

- Q - quadrature routine

- N - non-adaptive integrator
- A - adaptive integrator
- G - general integrand (user-defined)
- W - weight function with integrand
- S - singularities can be more readily integrated
- P - points of special difficulty can be supplied
- I - infinite range of integration
- O - oscillatory weight function, cos or sin
- F - Fourier integral
- C - Cauchy principal value

The algorithms are built on combination of quadrature rules, a lower order rule and a higher order rule. The higher order rule is used to compute the best approximation of the integral over a small range. The difference between the results of the higher order rule and the lower order rule gives an estimate of the error in the approximation.

The *gsl\_function* contains the value *x* as well as the parameters and is defined as

Listing 1.1: Definition of *gsl\_function* with Parameters

```

1 | struct gsl_function_struct {
   |     double (* function) (double x, void * params);
   |     void * params;
   | };
5 | typedef struct gsl_function_struct gsl_function ;
   | #define GSL_FN_EVAL(F,x) (*(F)->function)(x,(F)->params)

```

The integration region in the adaptive integration algorithms in *gsl* is divided into subintervals, and on each iteration the subinterval with the largest estimated error is bisected. This reduces the overall error rapidly, as the subintervals become concentrated around local difficulties in the integrand. These subintervals are managed by a *gsl\_integration\_workspace* struct, which handles the memory for the subinterval ranges, results and error estimates.

Function: *gsl\_integration\_workspace \* gsl\_integration\_workspace\_alloc (size\_t n\_max)*

This function allocates a workspace sufficient to hold *n\_max* double precision intervals, their integration results and error estimates.

Listing 1.2: workspace

```

1 | typedef struct {
   |     size_t limit;
   |     size_t size;
   |     size_t nrmax;
5 |     size_t i;
   |     size_t maximum_level;
   |     double *alist;

```

```

10  double *blist;
    double *rlist;
    double *elist;
    size_t *order;
    size_t *level;}
gsl_integration_workspace;

```

In *gsl\_workspace\_alloc* function, *n\_max* is the amount of memory allocated to workspace members *alist*, *blist*, *rlist*, *elist*, *order* and *level*.

## 2.1 Integrands Without Weight Functions

The algorithms for general functions (without a weight function) are based on Gauss-Kronrod rules. A Gauss-Kronrod rule begins with a classical Gaussian quadrature rule of order  $m$ . This is extended with additional points between each of the abscissae to give a higher order Kronrod rule of order  $2m + 1$ . The Kronrod rule is efficient because it reuses existing function evaluations from the Gaussian rule. The higher order Kronrod rule is used as the best approximation to the integral, and the difference between the two rules is used as an estimate of the error in the approximation.

## 2.2 Integrands With Weight Functions

For integrands with weight functions the algorithms use Clenshaw-Curtis quadrature rules. A Clenshaw-Curtis rule begins with an  $m$ -th order Chebyshev polynomial approximation to the integrand. This polynomial can be integrated exactly to give an approximation to the integral of the original function. The Chebyshev expansion can be extended to higher orders to improve the approximation and provide an estimate of the error.

## 2.3 Integrands With singular Weight Functions

The presence of singularities (or other behaviour) in the integrand can cause slow convergence in the Chebyshev approximation. The modified Clenshaw-Curtis rules used in QUADPACK separate out several common weight functions which cause slow convergence. These weight functions are integrated analytically against the Chebyshev polynomials to pre-compute modified Chebyshev moments. Combining the moments with the Chebyshev approximation to the function gives the desired integral. The use of analytic integration for the singular part of the function allows exact cancellations and substantially improves the overall convergence behaviour of the integration.

## 3 Algorithmic Differentiation of GSL Integration Routines

Algorithmic tangent and adjoint versions of the integration routine in *gsl* compute the directional derivatives of the approximation of the solution, which is actually computed by the algorithm [GW08,Nau12], in which AD is applied to the individual statements of the given implementation. In tangent mode, this yields an increase of roughly two in memory requirement as well as operation

count. In the adjoint mode, data required within the reverse section is recorded in the forward section. The resulting memory requirement is likely to exceed the available resources for most real-world applications <sup>1</sup>. In the adjoint version, the number of operations is two times the operations (OPS) performed by the algorithm itself. The required memory in this case is proportional to the number of operations.

In order to apply AD tool <sup>2</sup> to the gsl integration routines, a separate library *integration-multidim* is built according to *integration* library in gsl but with the following changes:

- Include *dco.hpp* in *gsl\_math.h* file, so that gsl knows the dco data types.
- Define the *gsl\_function* with dco types *dco :: gtls < double >:: type* and *dco :: ga1s < double >:: type* for tangent and adjoint version respectively.
- Define the related functions and routines with dco types.
- In some cases only the real value of the input is needed. In this case use the *get* function of dco. This returns the real part (double) of the input.

Note that gsl is written in C and dco is written in C++. In order to run dco in gsl, configure gsl with g++. For implementation set the right seed in the main function, call the integration routine and get the result of integration as well as the derivatives of the integral with respect to its parameters.

Listing 1.3: Algorithmic Tangent

```

1  gsl_integration_workspace_t1s_type* w =
      gsl_integration_workspace_alloc_t1s_type(100000);
struct my_f_params<dco::gtls<double>::type> params;
for(int i=0; i<n; i++) {
5     params.alpha = vec_alpha;
      dco::gtls<double>::set(params.alpha[i], 1., 1);
      initialise_boundaries(a, b, params.alpha);
      gsl_function_t1s_type F;
      F.function = &func;
10     F.params = &params;
      gsl_integration_qags(&F, a, b, 1e-7, 1e-7, w->limit,
                          w, &result, &error);
      dco::gtls<double>::get(result, presult);
      dco::gtls<double>::get(result, dresult, 1);
15     printf(" dI/da[%d]=%f\n", i, dresult);
      sum_deriv += dresult;}
      printf(" Diff. of Integration:%f\n", sum_deriv);
      printf(" The integration: %f\n", presult);
      gsl_integration_workspace_free(w); }

```

In listing 1.3 the algorithmic tangent mode of AD is used to differentiate the integration of a gsl function (listing 1.1), which in the above listing is evaluated

<sup>1</sup> Checkpointing techniques can help keeping the required memory feasible at the expense of additional function evaluations, See [Gri92]. for details.

<sup>2</sup> AD tools are: dco (Derivative Code by Overloading) and dcc (Derivative Code Compiler). In this paper we apply dco as AD tool.

with qags routine. For this purpose, a workspace of size 100000 is defined. The function has  $n$  parameters and the boundaries  $a, b \in \mathbb{R}$  are dependent to the parameters. Differentiating this function with respect to all parameters with algorithmic tangent, a loop of size  $n$  is defined in line 4. After setting the function with its parameters, the integration routine is called in line 11. Furthermore, with every call of the integration routine (i.e. for each  $\alpha_i, i = 1, \dots, n$ ),  $\nu_1$  number of iterations will be applied in order to approximate the integral. In algorithmic tangent mode,  $\nu_1$  in every call of the integration routine is the same.

Listing 1.4: Algorithmic Adjoint

```

1  gsl_integration_workspace_als_type* w =
    gsl_integration_workspace_alloc_als_type(100000);
struct my_f_params<dco::gals<double>::type> params;
    params.alpha = vec_alpha;
5  dco::gals<double>::global_tape
    -> register_variable(params.alpha);
    initialise_boundaries(a, b, params.alpha);
    gsl_function_als_type F;
    F.function = &func;
10  F.params = &params;
    gsl_integration_qags(&F, a, b, 1e-7, 1e-7, w->limit,
        w, &result, &error);
    dco::gals<double>::set(result, 1, -1);
    dco::gals<double>::get(result, presult);
15  dco::gals<double>::global_tape->interpret_adjoint();
    for(int ii=0; ii<n; ii++) {
        dco::gals<double>::get(params.alpha[ii], aux, -1);
        printf("dI/da[%d]=%f\n", ii, aux);
        dresult += aux;}
20  dco::gals<double>::global_tape->reset();
    printf("Diff. of Integration: %f\n", dresult);
    printf("The integration: %f\n", presult);
    gsl_integration_workspace_free(w); }

```

In listing 1.4 the algorithmic adjoint mode of AD is used to differentiate the integration of a gsl function. The same as tangent mode, a workspace of size 100000 is defined, the function has  $n$  parameters and the boundaries  $a, b \in \mathbb{R}$  are dependent to the parameters. Differentiating this function with respect to all parameters with algorithmic adjoint, the parameters should be registered in tape for backward interpretation. After setting the function with its parameters, the integration routine is called just once in line 11, and like algorithmic tangent mode, by the call of integration routine,  $\nu_1$  iterations will be applied in order to approximate the integral. With one interpretation we evaluate the integral as well as the derivative of the integration routine with respect to all parameters.

Computational complexity of  $n$  projections with algorithmic tangent and adjoint modes for differentiating the gsl integration routine with  $\nu_1$  iterations is  $\nu_1 \cdot O(n)$  and  $\nu_1 \cdot O(1)$  respectively, and the memory requirement of algorithmic adjoint mode for  $n$  projections is  $\nu_1 \cdot O(n)$ .



## 4 Symbolic Differentiation of GSL Integration routines

The symbolic differentiation of the integral  $I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} f(\boldsymbol{\alpha}, x) dx$  with respect to  $\boldsymbol{\alpha}$  is evaluated by computing Equation (4) with  $\boldsymbol{\alpha} \in \mathbb{R}^n$ . In order to evaluate the derivatives in the symbolic mode we apply AD tool, it means the evaluation of  $\nabla a$ ,  $\nabla b$  and  $\nabla f$  are done with dco. After computing the derivatives with AD, the integration routine can be called with its original data type *double*. There are two possibilities to compute the derivatives in dco, either with tangent mode AD or with adjoint mode AD. Evaluating the derivatives with tangent mode AD and then integrating the function is straightforward.

Listing 1.5: Function Wrapper Tangent

```

1  double f_wrapper_t1(double x, void *params){
    struct my_f_params<dco_t1_type> param_alpha =
        *(struct my_f_params<dco_t1_type> *)params;
    dco_t1_type x_active = x;
5   dco_t1_type prod;
    dco::gtls<double>::set
        (param_alpha.alpha[indx], 1., 1);
    prod = func(x_active, &param_alpha);
    double derivative = 0;
10  dco::gtls<double>::get(prod, derivative, 1);
    dco::gtls<double>::set
        (param_alpha.alpha[indx], 0, 1);
    return derivative;}

```

According to Equation (4), computing the derivatives of the integral with symbolic mode, the differentiation of the function should be passed to the integration routine as integrand instead of the function itself. The above implementation defines the differentiation (with tangent mode) of the function which should be integrated, with respect to one parameter, i.e.  $dF_Tg = \left(\frac{\partial f(\boldsymbol{\alpha}, x)}{\partial \alpha_{indx}}\right)$ , where  $indx \in [1, n]$ . The output is scalar and this function is actually the integrand in the symbolic tangent mode.

Listing 1.6: Symbolic Tangent

```

1  gsl_integration_workspacedouble* w =
    gsl_integration_workspace_allocdouble(100000);
    struct my_f_params<double> cont_params;
    for(int i=0; i<n; i++) {
5   params.alpha = vec_alpha;
    dco::gtls<double>::set(params.alpha[i], 1., 1);
    indx = i;
    initialise_boundaries(a, b, params.alpha);
    dco::gtls<double>::get(a, pa);
10  dco::gtls<double>::get(b, pb);
    dco::gtls<double>::get(a, da, 1);
    dco::gtls<double>::get(b, db, 1);
    pre_result = func(pb, &cont_params)*db
        - func(pa, &cont_params)*da;

```

```

15     gsl_functiondouble dF_Tg;
        dF_Tg.function = &f_wrapper_t1;
        dF_Tg.params = &params;
        gsl_integration_qags(&dF_Tg, pa, pb, 1e-7, 1e-7,
                               w->limit, w, &result, &error);
20     dco::gtls<double>::set(params.alpha[i], 0, 1);
        cont_integral[i] = pre_result + result;
        printf("dI/da[%d]=%f\n", i, cont_integral[i]);
        sum_deriv += cont_integral[i]; }
25     printf("Diff. of Integration:%f\n", sum_deriv);
        gsl_integration_workspace_free(w); }

```

Listing 1.6 is defined to differentiate the integration of a gsl function with symbolic tangent mode. The function has  $n$  parameters and the boundaries  $a$  and  $b$  are dependent to the parameters. Differentiating this function with respect to all parameters with symbolic tangent, a loop of size  $n$  is defined in line 4, which implies the integration routine should be called  $n$  times. Furthermore, with every call of the integration routine,  $\nu_2$  number of iterations will be applied in order to approximate the integral. In symbolic tangent mode, the number of  $\nu_2$  iterations in every call of the integration routine can be different. This is because the integrand is the differentiation of the gsl function  $\left(\frac{\partial f(\boldsymbol{\alpha}, x)}{\partial \alpha_i}\right), i = 1, \dots, n$ , which can be different for each  $i$ .

The differences between this evaluation with the one in listing 1.3 are: in the above implementation, the data type of the variables in the integration routine as well as in gsl function is *double*, whereas in algorithmic tangent they are of *dco :: gtls < double >:: type* type, the function which is passed to the integration routine in symbolic tangent is the differentiation of the function which should be passed to the integration routine in algorithmic tangent, additionally *pre\_result =  $\nabla b f(\boldsymbol{\alpha}, b) - \nabla a f(\boldsymbol{\alpha}, a)$*  should be evaluated.

For some routines in gsl, the function which is defined to be integrated differs from the original function which should be integrated. For example, suppose a function which is defined as  $f(\boldsymbol{\alpha}, x) = \sum_{i=1}^{i=n} \frac{\sin(\alpha_i \cdot x)}{\alpha_i^2}$ . Applying *gsl\_integration\_qawc* routine (which is an integration routine for integrating the functions with a singularity at  $c$  and  $c \in (a, b)$ ) on it, then gsl considers this function as  $f(\boldsymbol{\alpha}, x) = \sum_{i=1}^{i=n} \frac{\sin(\alpha_i \cdot x)}{\alpha_i^2 \cdot (x-c)}$  (which in this paper we call the original function), just because of applying *gsl\_integration\_qawc* on it. Differentiating the integrals with symbolic mode, it should be noticed that for computing  $f(a, \boldsymbol{\alpha})$  and  $f(b, \boldsymbol{\alpha})$  the original function should be considered as  $f$ .

Evaluating the derivatives with adjoint mode AD and then integrating it, is tricky, because in adjoint mode in case of scalar output, with one function call we get the derivative of the output of that function with respect to all inputs.

As it is shown in Equation 4, in order to evaluate the derivatives of the integral with respect to its parameters with symbolic mode, instead of the integrand, the derivative of the integrand with respect to parameters should be integrated.

Differentiating the *gsl\_function* with adjoint mode AD in order to calculate  $\nabla f$ , the output will not be scalar any more, but a vector of size  $n$ . According to this reason, a vectorized version of gsl integration routines should be defined. For this purpose, we build a new library e.g. *integration-multidim*, in which the dimension  $n$  should be added to the structure of *gsl\_function*. Therefore, we define a *gsl\_function\_vec* as

Listing 1.7: Definition of *gsl\_function\_vec* with Parameters

```

1 struct gsl_function_vec_struct {
    int dim;
    std::vector<double> (* function)
                                int dim, double x, void * params);
5 void * params;
};
typedef struct gsl_function_vec_struct gsl_function_vec;
#define GSL_FN_VEC_EVAL(F,x)
                                (*( (F)->function ))( (F)->dim, x, (F)->params)

```

Hence, the whole routines, classes, structures and functions should be changed in a way that they can deal with a vector function (and not scalar function as default). In this case, the evaluation of all of the results (i.e. the differentiation of the integral with respect to all parameters) and all of the respective absolute errors are done simultaneously, therefore, *result* and *abserr* (which are outputs) in the integration routines, should be defined as vectors.

The value *n\_max* in workspace determines the maximum number of bisections and as result the maximum number of approximations of the results and absolute errors in the interval. The adaptive integration routines in gsl iterate and bisect the integration region until reach to the tolerance. For the cases that we need more iterations (bisections) of the integral region than *n\_max*, we get a *GSL\_ERROR* : *the number of iterations was insufficient to reach the tolerance*. By using the adjoint mode AD in the symbolic version the dimension of *rlist* and *elist* in Listing 1.2 should be increased to  $n\_max \times n$  (instead of *n\_max*), because the approximation of the integral for our integrand  $\nabla f$  as well as the absolute error estimates for all  $\alpha_i, i = 1, \dots, n$  will be done at the same time. Allocating  $n\_max \times n$  memory to *result* and *abserr* especially for cases that we need significantly less iterations than *n\_max* is not efficient. Therefore, we allocate at first  $n$  units of memory to them and with every bisection we increase the size of allocated memory by 1. The dimensions of other *workspace* members stay the same.

In the adaptive routines of gsl integration routines the error estimates are compared and the interval with the largest error is bisected. What should we do now that we have  $n$  error estimates for each interval? The answer is, in this paper, we compare the  $n$  error estimates and determine the maximum one on each interval and the interval with the largest error would be bisected. It results that, at the end the number of iterations performed by the routine is nearly equal to the largest number of iterations performed by symbolic tangent for each parameter.

Listing 1.8: Function Wrapper Adjoint

```

1  std::vector<double> f_wrapper_al
      (int n, double x, void *params) {
      struct my_f_params<dco_alm_type> param_alpha =
          *(struct my_f_params<dco_alm_type> *)params;
5  ad_mode::tape_t_options options;
      options.chunksize() = 10*alpha_dim;
      static ad_mode::tape_t *tape =
          ad_mode::tape_t::create(options);
      dco_alm_type x_active = x, prod;
10  std::vector<double> deriv(n, 0);
      tape->register_variable(param_alpha.alpha);
      prod = func(x_active, &param_alpha);
      ad_mode::set(prod, 1., -1);
      tape->interpret_adjoint();
15  ad_mode::get(param_alpha.alpha, deriv, -1);
      tape->reset();
      return deriv;}

```

Listing 1.8 defines the differentiation (with adjoint mode) of the function which should be integrated  $dF_{Adj} = \nabla f$ . This function is actually the integrand in the symbolic adjoint mode. As shown in lines 6–7 in Listing 1.8, a local tape of size  $(10 \times n)$  is defined to store the intermediate variables for the reverse interpretation in order to evaluate  $dF_{Adj}$ .

Listing 1.9: Symbolic Adjoint

```

1  gsl_integration_workspace* w =
      gsl_integration_workspace_alloc(100000, n);
      std::vector<double> result(n), error(n);
      struct my_f_params<dco::gals<double>::type> params;
5  params.alpha = vec_alpha;
      struct my_f_params<double> cont_params;
      dco::gals<double>::global_tape->
          register_variable(params.alpha);
      initialise_boundaries(a, b, params.alpha);
10  dco::gals<double>::global_tape->
          register_output_variable(a);
      dco::gals<double>::global_tape->
          register_output_variable(b);
      dco::gals<double>::set(a, 1, -1);
15  dco::gals<double>::global_tape->interpret_adjoint();
      dco::gals<double>::get(a, pa);
      dco::gals<double>::get(params.alpha, deriv_a, -1);
      dco::gals<double>::global_tape->zero_adjoints();
      dco::gals<double>::set(b, 1, -1);
20  dco::gals<double>::global_tape->interpret_adjoint();
      dco::gals<double>::get(b, pb);
      dco::gals<double>::get(params.alpha, deriv_b, -1);
      cont_params.alpha = glob_vec_alpha;

```

```

25  aux0 = func(pb, &cont_params);
    aux1 = func(pa, &cont_params);
    for(int i=0; i<n; i++)
        pre_result[i] = aux0*deriv_b[i] - aux1*deriv_a[i];
    d_params.alpha = vec_alpha_d;
    gsl_function_vec dF_Adj;
30  dF_Adj.dim = n;
    dF_Adj.function = &f_wrapper_a1;
    dF_Adj.params = &d_params;
    gsl_integration_qags(&dF_Adj, pa, pb, 1e-7, 1e-7,
                        w->limit, w, result, error);
35  for(int i=0; i<n; i++) {
        printf(dI/da[%d]=%f \n", i, pre_result[i]+result[i]);
        sum_deriv += pre_result[i] + result[i];
    }
    dco::gals<double>::global_tape->reset();
    printf("Diff. of Integration:%f \n", sum_deriv);
40  gsl_integration_workspace_free(w);

```

Listing 1.9 is defined to differentiate the integration of a gsl function with symbolic adjoint mode. For this purpose, the dimension of the parameters ( $n$ ) is added to the structure of *workspace\_alloc* in order to allocate memory of  $n$  to *rlist* and *elist* in the workspace (listing 1.2). The outputs *result* and *abserr* are defined as vectors of size  $n$ . The gsl function is here a vector function (listing 1.7). The vector function which should be passed to the integration routine is the adjoint differentiation of the integrand (listing 1.8). After setting the function with its parameters and computing  $pre\_result = \nabla b f(\boldsymbol{\alpha}, b) - \nabla a f(\boldsymbol{\alpha}, a)$ , the integration routine is called just once in line 33, and by the call of integration routine,  $\nu_2$  iterations will be applied in order to approximate the integral. In the symbolic adjoint mode, the number of  $\nu_2$  iterations is nearly equal to the maximum  $\nu_2$  number of iterations in symbolic tangent.

Computational complexity of  $n$  projections with symbolic tangent and adjoint modes for differentiating the gsl integration routine (e.g. qags) with  $\nu_2$  iterations is  $\nu_2 \cdot O(n)$  and  $\nu_2 \cdot O(1)$  respectively. The memory requirement of symbolic adjoint mode for  $n$  projections is  $O(n)$ , which contains the memory requirement for evaluating  $\nabla f$ , that in this paper is defined to be  $(10 \times n)$ , and the memory requirement of computing  $\nabla a$  and  $\nabla b$ , that is also  $O(n)$ .

## 5 Test Cases

This chapter describes and compares routines for performing numerical integration (quadrature) of a function with multi dimensional parameters and the differentiation of the integration with different methods, i.e. algorithmic tangent/adjoint and symbolic tangent/adjoint. It is important to choose a function as test case, in which the corresponding integration routine is suitable for that test case and also the same integration routine is suitable for the differentiation of that function, because in this paper we use the same integration routine for both symbolic and algorithmic modes. The duration of the computation depends strongly on the number of iterations performed by the integration routine and

the number of iteration depends on the integrand and the specified accuracy. As mentioned in the previous section, the number of iterations for algorithmic and symbolic computation can differ, because in algorithmic version, the integrand is the function, however, in symbolic mode, the integrand is the derivative of the function with respect to its parameters. Furthermore, the number of iterations in symbolic tangent differentiation can be different for each parameter, but with applying symbolic adjoint, we have just one number of iterations, which is nearly the same as maximum number of iterations applied by symbolic tangent. In this section, in case of illustrating the number of iterations with symbolic mode, we consider the number of iterations applied by symbolic adjoint .

All of the following measurements are done on a machine with 2x Intel(R) Xeon(R) CPU E5-2630 0 @ 2.30GHz (2x 6 Cores (12 Threads)), 128 GB RAM.

1. **QAG adaptive integration:** The QAG algorithm is a simple adaptive integration procedure. The adaptive functions apply an integration rule adaptively until an estimate of the integral of  $f$  over  $(a, b)$  is achieved within the desired absolute and relative error limits,  $epsabs$  and  $epsrel$ . The function returns the final approximation,  $result$ , and an estimate of the absolute error,  $abserr$ .

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \sum_{i=1}^n \frac{\cos(\alpha_i^3 \cdot x)}{x} dx \quad .$$

where  $a(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i$  and  $b(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qag routine. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead is shown in Figure 1 and in Table 1 the memory requirement as well as number of iterations are illustrated.

n	Symbolic		Algorithmic		Finite Diff
	Tg	Adj	Tg	Adj	
5	0.43	0.16	0.4	0.16	0.43
10	1.17	0.56	2.11	0.44	2.25
15	3.34	0.8	6.19	0.84	6.55
20	8.16	2.08	21.52	2.21	23.55
50	91.79	10.08	266.4	10.8	286.12
80	-	31.85	-	34.21	-

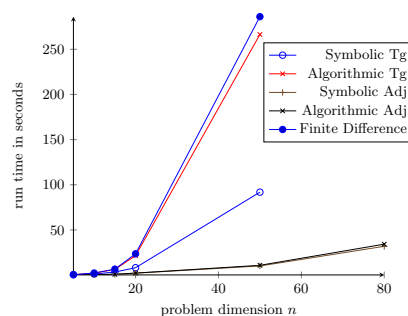


Fig. 1: Run time overhead in seconds for qag routine. Missing values indicate failure to converge within 300 seconds.

In this test case, absolute error is set as well as relative error to  $10^{-7}$ . To reach this accuracy, e.g. for  $n = 10$ , the number of iterations applied by symbolic and algorithmic is 2041 and 1523 respectively. As illustrated in Figure 1, evaluating derivatives with adjoint modes is considerably more efficient in terms

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
5	0.001	1024	97.02	1024
10	0.002	2041	250.62	1523
15	0.003	2048	479.96	2048
20	0.004	4096	1233.82	4056
50	0.01	8192	5923.07	8192
80	0.018	16384	18647.4	16330

Table 1: Memory Requirement in MB and number of iterations ( $\nu$ ) for qag routine.

of runtime than applying tangent modes. For this function with this accuracy, symbolic tangent is faster than algorithmic tangent, whereas symbolic and algorithmic adjoint have nearly the same runtime overhead. However, the memory requirement of algorithmic adjoint is significantly higher than the one for symbolic adjoint.

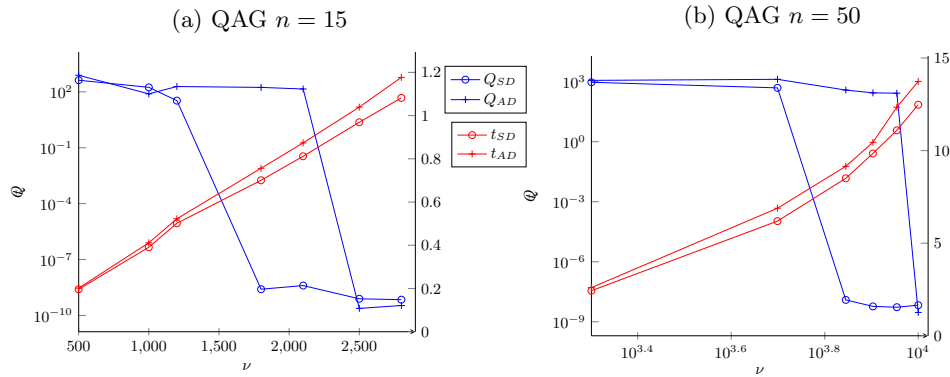


Fig. 2: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qag routine using different approaches to differentiation.

Figure 2 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different number of iterations  $\nu$  using different differentiation methods in the integration of the reference problem with qag routine for  $n = 15$  and  $n = 50$ . In this section the discrepancy between the adjoints computed in  $\nu$ th iteration of the integration routine with its value in the previous iteration yields:

$$Q = \|\alpha_{(1)\nu_j} - \alpha_{(1)\nu_{(j-1)}}\| \quad , \quad j \in (1, n_{max}) \quad . \quad (5)$$

Suppose the convergence  $\delta = 10^{-8}$ , Figure 2 shows that to reach this accuracy, algorithmic adjoint requires  $\nu = 2500$  and  $\nu = 10000$  iterations whereas symbolic adjoint needs  $\nu = 1800$  and  $8000$  iterations for  $n = 15$  and  $n = 50$  respectively. The behaviour of symbolic and algorithmic adjoints in terms of runtime overhead is nearly the same, however symbolic adjoint is a bit faster.

- 2. QAGS adaptive integration with singularities:** The presence of an integrable singularity in the integration region causes an adaptive routine to concentrate new subintervals around the singularity. As the subintervals decrease

in size the successive approximations to the integral converge in a limiting fashion. This approach to the limit can be accelerated using an extrapolation procedure. The QAGS algorithm combines adaptive bisection with the Wynn epsilon-algorithm to speed up the integration of many types of integrable singularities.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \sum_{i=1}^n \frac{\alpha_i^3 \cdot \sin(x)}{x} dx \quad .$$

where  $a(\boldsymbol{\alpha}) = -\sum_{i=1}^n \alpha_i$  and  $b(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qags routine. This function has a singularity in  $x = 0$ . Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 3 and Table 3 respectively.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.01	0.01	0.01	0.004	0.01
50	0.4	0.04	0.31	0.04	0.28
100	2.3	0.14	2.01	0.19	2.05
300	51.45	1.19	50.49	1.63	47.05
500	203.88	2.81	191.17	3.8	180.11
1000	-	11.74	-	15.1	-
1500	-	32.66	-	42.47	-

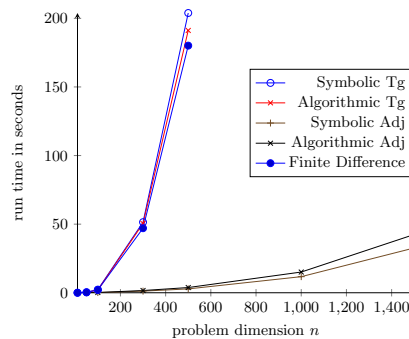


Fig. 3: Run time overhead in seconds for qags routine. Missing values indicate failure to converge within 300 seconds.

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
10	0.002	31	1.75	31
50	0.01	128	31.82	128
100	0.02	254	124.18	254
300	0.07	722	1047.01	722
500	0.1	1022	2464.34	1022
1000	0.2	2043	9836.12	2043
1500	0.3	3880	28006	3880

Table 2: Memory Requirement in MB and number of iterations ( $\nu$ ) for qags routine.

In this test case, absolute error is set as well as relative error to  $10^{-7}$ . To reach this accuracy, e.g. for  $n = 100$ , the number of iterations for both symbolic and algorithmic is 254. As illustrated in Figure 3, for this function with this accuracy, algorithmic tangent is faster than symbolic tangent, whereas symbolic adjoint is faster than algorithmic adjoint. Furthermore, the memory



requirement of algorithmic adjoint is considerably higher than the memory requirement of the symbolic adjoint.

Figure 4 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different  $\nu$ s using different differentiation methods in the integration of the reference problem with qags routine and shows that symbolic adjoint converges faster in comparison with algorithmic adjoint, e.g. for  $\delta = 10^{-10}$  and  $n = 300$ , symbolic adjoint needs  $\nu = 722$  and algorithmic adjoint requires  $\nu = 1022$  iterations. Furthermore, the time spent by symbolic adjoint is less than time spent by algorithmic adjoint. By increasing  $n$  and  $\nu$ , the difference between duration of computation by adjoint algorithmic and symbolic becomes larger.

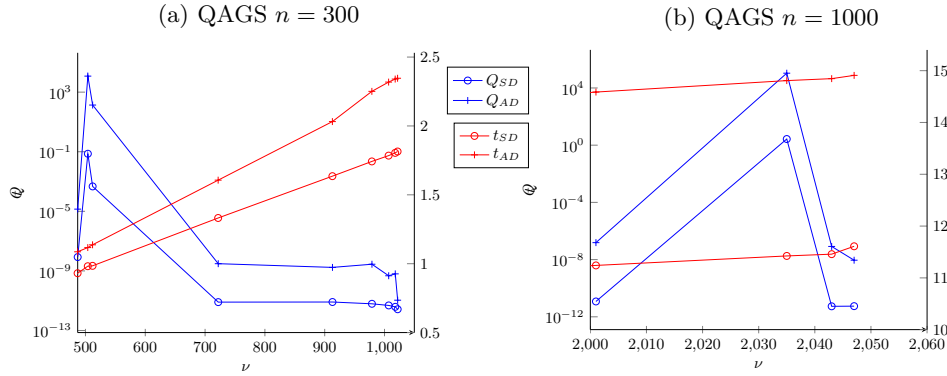


Fig. 4: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qags routine using different approaches to differentiation.

As mentioned before, we should take care of choosing right integrand in order to be able to apply the same integration routine on the corresponding integrand for both symbolic and algorithmic differentiation. For example, suppose a function  $f(\alpha, x) = \sum_{i=1}^n \frac{\sin(\alpha_i^3 \cdot x)}{x}$  which has singularity at  $x = 0$ , therefore the integration routine qags can be applied on it, but the differentiation of the function  $\nabla f = \sum_{i=1}^n 2\alpha_i^2 \cos(\alpha_i^3 \cdot x)$  which will be the integrand by using symbolic differentiation has no singularity at  $x = 0$ , therefore applying qags routine on it is not efficient.

3. **QAGI adaptive integration on infinite intervals:** This algorithm uses the QAGS algorithm, which computes the integral of the function  $f$  over the infinite interval  $(-\text{inf}, +\text{inf})$ . The integral is mapped onto the semi-open interval  $(0, 1]$  using the transformation  $x = (1 - t)/t$ .
4. **QAGIU adaptive integration with infinite upper boundary:** This algorithm uses the QAGS algorithm, which computes the integral of the function  $f$  over the semi-infinite interval  $(a, +\text{inf})$ . The integral is mapped onto the semi-open interval  $(0, 1]$  using the transformation  $x = a + (1 - t)/t$ .
5. **QAGIL adaptive integration with infinite upper boundary:** This algorithm uses the QAGS algorithm, which computes the integral of the func-

tion  $f$  over the semi-infinite interval  $(-\inf, b)$ . The integral is mapped onto the semi-open interval  $(0, 1]$  using the transformation  $x = b - (1 - t)/t$ .

6. **QAWC adaptive integration with one singularity at  $x=c$** : This function computes the Cauchy principal value of the integral of  $f$  over  $(a, b)$ , with a singularity at  $c$ ,  $I = \int_a^b dx f(x)/(x - c)$ . The adaptive bisection algorithm of QAG is used, with modifications to ensure that subdivisions do not occur at the singular point  $x = c$ . When a subinterval contains the point  $x = c$  or is close to it then a special 25-point modified Clenshaw-Curtis rule is used to control the singularity. Further away from the singularity the algorithm uses an ordinary 15-point Gauss-Kronrod integration rule.

This routine is used by integrands with weight functions and for evaluation of integrals with this method, table of chebyshev moments in every iteration of the integration routine should be computed. For this purpose, there exist two variables: Cheb12 and Cheb24 of size 13 and 25 respectively. In the vectorized version of gsl, the size of Cheb12 and Cheb24 should be increased by factor of  $n$  in order to make the simultaneous computation for all parameters possible.

As case study, we consider evaluating the differentiation of the integral

$$I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} \sum_{i=1}^n \frac{\sin(\alpha_i^3 \cdot x)}{(x - c)} dx \quad ,$$

where  $c \in (a, b)$ ,  $a(\alpha) = -\sum_{i=1}^n \alpha_i$  and  $b(\alpha) = \sum_{i=1}^n \alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qawc routine. This function has a singularity in  $c \in (a, b)$  in both symbolic and algorithmic versions. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 5 and Table 5 respectively.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
5	1.29	0.76	2.05	0.73	2.84
10	4	1.34	7.98	1.36	9.97
20	28.57	8.46	101.23	7.72	133.81
30	67.27	15.29	273.66	14.85	390.26
35	99.18	17.28	361.85	17.17	510.17

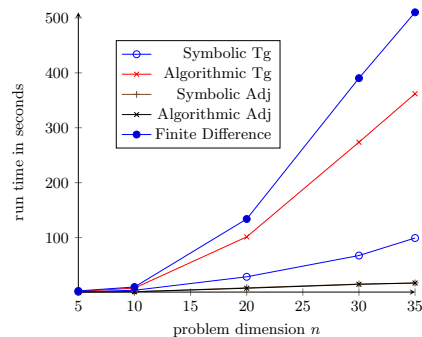


Fig. 5: Run time overhead in seconds for qawc routine.

In this test case, absolute error is set as well as relative error to  $10^{-7}$ . To reach this accuracy, e.g. for  $n = 10$ , the number of iterations for symbolic and algorithmic is 16614 and 17906 respectively. As illustrated in Figure 5, for this function with this accuracy, symbolic tangent is faster than algorithmic tangent, whereas the behaviour of adjoint symbolic and algorithmic in terms

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
5	0.001	15860	368.34	15693
10	0.002	16614	686.17	17906
20	0.004	48034	3276.37	48109
30	0.007	64540	6269.02	64065
35	0.008	66374	7460.27	66198

Table 3: Memory Requirement in MB and number of iterations ( $\nu$ ) for qawc routine.

of runtime is the same. However, the memory requirement of algorithmic adjoint is considerably higher than the memory requirement of the symbolic adjoint.

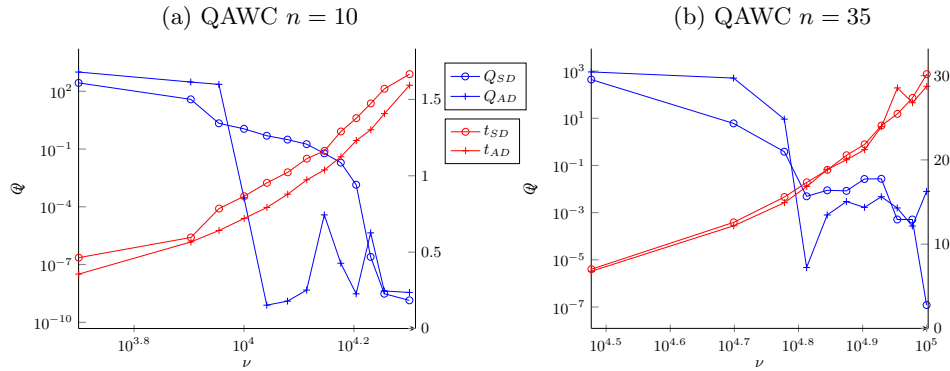


Fig. 6: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qawc routine using different approaches to differentiation.

Figure 6 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different  $\nu$ s using different differentiation methods in the integration of the reference problem with qawc routine and shows that at first ( $\nu < 11000$  and  $\nu < 65000$  for  $n = 15$  and  $n = 35$  respectively) symbolic adjoint converges faster in comparison with algorithmic adjoint, but after that the convergence of algorithmic adjoint gets more speed. For  $n = 10$  adjoint algorithmic spends less time, but for  $n = 35$  the time spent for both methods is nearly the same.

#### 7. QAWS adaptive integration for functions with singular endpoints:

The QAWS algorithm is designed for integrands with algebraic-logarithmic singularities at the end-points of an integration region. In order to work efficiently the algorithm requires a precomputed table of Chebyshev moments. The adaptive bisection algorithm of QAG is used. When a subinterval contains one of the endpoints then a special 25-point modified Clenshaw-Curtis rule is used to control the singularities. For subintervals which do not include the endpoints an ordinary 15-point Gauss-Kronrod integration rule is used. This routine is used by integrands with weight functions and for evaluation of integrals with this method, table of chebyshev moments in every iteration of the integration routine should be computed.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a=0}^{b=1} \sum_{i=1}^n \frac{\cos(\alpha_i^2) \cdot (\log(\alpha_i \cdot x))^2}{(1-x)^2 \cdot \alpha_i^2} dx \quad .$$

with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qaws routine. This function is singular in endpoints  $a$  and  $b$ , therefore, the endpoints should not depend on  $\alpha$ , because in case of dependent boundaries in symbolic differentiation of integrals,  $f(a, \alpha)$  and  $f(b, \alpha)$  should be computed, which in this case do not exit because of the singularity. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 7 and Table 4 respectively.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.05	0.01	0.04	0.01	0.04
100	2.68	0.06	2.58	0.05	3.2
500	71.04	0.29	72.29	0.28	87.62
700	138.68	0.41	141.5	0.40	172.37
1000	-	0.74	-	0.6	-
5000	-	3.19	-	2.88	-
10000	-	6.27	-	5.77	-
20000	-	12.53	-	11.6	-
50000	-	31.83	-	28.92	-
100000	-	65.27	-	57.84	-

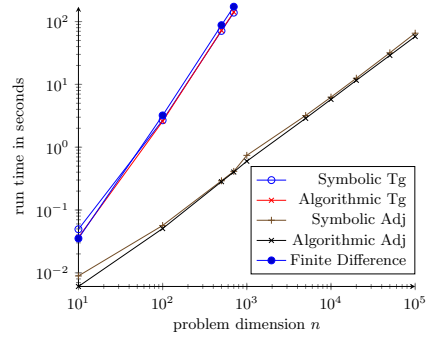


Fig. 7: Run time overhead in seconds for qaws routine. Missing values indicate failure to converge within 300 seconds.

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
10	0.002	49	3.61	46
100	0.02	50	31.97	49
500	0.09	50	157.06	49
700	0.12	50	219.6	49
1000	0.18	50	313.41	49
5000	0.89	50	1564.24	49
10000	1.75	50	3127.77	49
20000	3.51	50	6254.83	49
50000	8.77	50	15636	49
100000	17.55	50	31271.4	49

Table 4: Memory Requirement in MB and number of iterations ( $\nu$ ) for qaws routine.

In this test case, absolute error is set as well as relative error to  $10^{-12}$ . To reach this accuracy, e.g. for  $n = 500$ , the number of iterations for symbolic and algorithmic is 50 and 49 respectively. As illustrated in Figure 7, for this function with this accuracy, for small  $n$  tangent methods as well as adjoint

methods have the same runtime overhead, but by increasing the size  $n$ , algorithmic adjoint spends less time compared to other differentiation methods. The memory requirement of algorithmic adjoint is significantly higher than the memory requirement of symbolic adjoint.

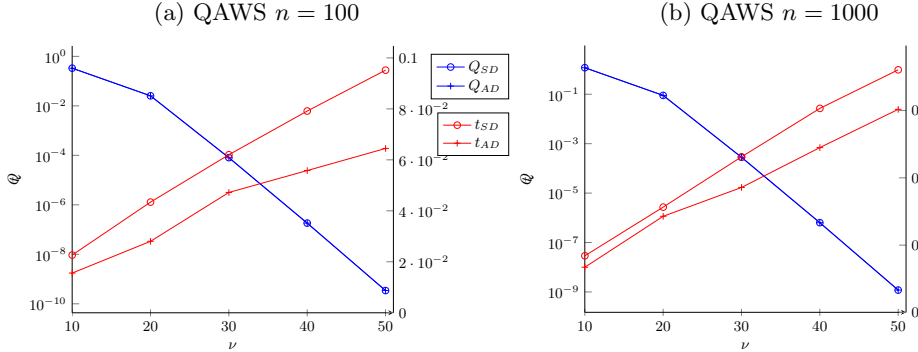


Fig. 8: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qaws routine using different approaches to differentiation.

Figure 8 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different  $\nu$ s using different differentiation methods in the integration of the reference problem with qaws routine. Because of independence of the boundaries to parameters, in Equation (4) we have  $\nabla b = 0$  and  $\nabla a = 0$ . It results the same behaviour in terms of convergence for both adjoint algorithmic and symbolic methods. However, because of computation of the Chebyshev table of moments in qaws and increasing the size of it in the symbolic adjoint mode, the symbolic adjoint methods spends more time than algorithmic adjoint method. The difference of duration for symbolic and algorithmic for  $n = 100$  is  $\approx 0.03$  and for  $n = 1000$  is  $\approx 0.12$  seconds.

8. **QAWO adaptive integration for oscillatory functions:** This algorithm is designed for integrands with an oscillatory factor,  $\sin(\omega x)$  or  $\cos(\omega x)$ . In order to work efficiently the algorithm requires a table of Chebyshev moments which must be pre-computed. Those subintervals with *large* widths where  $d\omega > 4$  are computed using a 25-point Clenshaw-Curtis integration rule, which handles the oscillatory behavior. Subintervals with a *small* widths where  $d\omega < 4$  are computed using a 15-point Gauss-Kronrod integration. This routine is used by integrands with weight functions and for evaluation of integrals with this method, table of chebyshev moments in every iteration of the integration routine should be computed. QAWF routine (see below) uses QAWO in the computation of integrals.

9. **QAGP adaptive integration with known singular points:** This function applies the adaptive integration algorithm QAGS taking account of the user-supplied locations of singular points. The array  $pts$  of length  $npts$  should contain the endpoints of the integration ranges defined by the integration region and locations of the singularities. If you know the locations of the singular points in the integration region then this routine will be faster than QAGS.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \sum_{i=1}^n \alpha_i^3 \cdot x^3 \log \left( \frac{|(x^3 - p_1^3) \cdot (x^2 - p_2^2)|}{\alpha_i + 1} \right) dx \quad .$$

where  $p_1, p_2 \in (a, b)$ ,  $p_1 < p_2$ ,  $a(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i$  and  $b(\boldsymbol{\alpha}) = 4 * \sum_{i=1}^n \alpha_i$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qagp routine. This function is singular in  $x = p_1$  and  $x = \pm p_2$ , however,  $x = -p_2$  is not in our integration region. Therefore, we have 2 singular points.

```
std::vector<double> pts(4, 0);
pts[0] = a;
pts[1] = p1;
pts[2] = p2;
pts[3] = b;
gsl_integration_qagp (&f, pts, n, 1e-7, 1e-7, w->limit,
                    w, &result, &abserr) ;
```

Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 9 and Table 5 respectively.

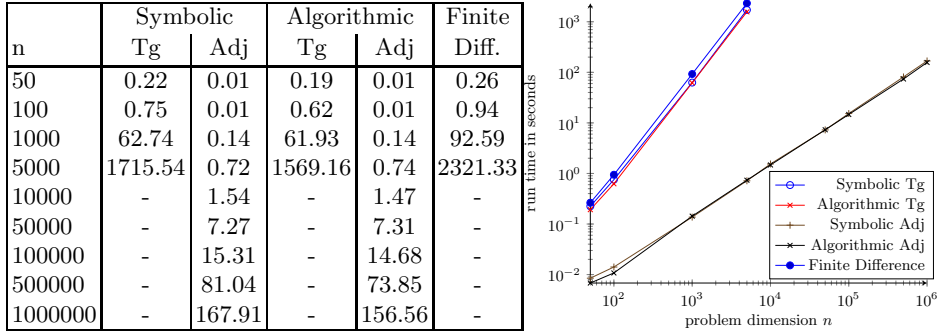


Fig. 9: Run time overhead in seconds for qagp routine. Missing values indicate failure to converge within 3000 seconds.

In this test case, absolute error is set as well as relative error to  $10^{-7}$ . To reach this accuracy, e.g. for  $n = 5000$ , the number of iterations for both symbolic and algorithmic is 21. As illustrated in Figure 9, for this function with this accuracy, algorithmic and symbolic adjoint methods have nearly the same runtime overhead, but the memory requirement of algorithmic adjoint is significantly higher.

Figure 10 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different  $\nu$ s using different differentiation methods in the integration of the reference problem with qagp routine. It shows that, for  $n = 100$  and different number of iterations, algorithmic adjoint spends less time, however algorithmic and symbolic adjoint have nearly the same runtime behaviour for  $n = 1000$ . The convergence of both methods are nearly the same.

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
50	0.01	20	4.88	21
100	0.02	21	9.56	21
1000	0.21	21	93.72	21
5000	1.03	21	467.81	21
10000	2.06	21	935.42	21
50000	10.3	21	4676.28	21
100000	20.6	21	9352.33	21
500000	103	21	46760.8	21
1000000	206	21	93521.4	21

Table 5: Memory Requirement in MB and number of iterations ( $\nu$ ) for qagp routine.

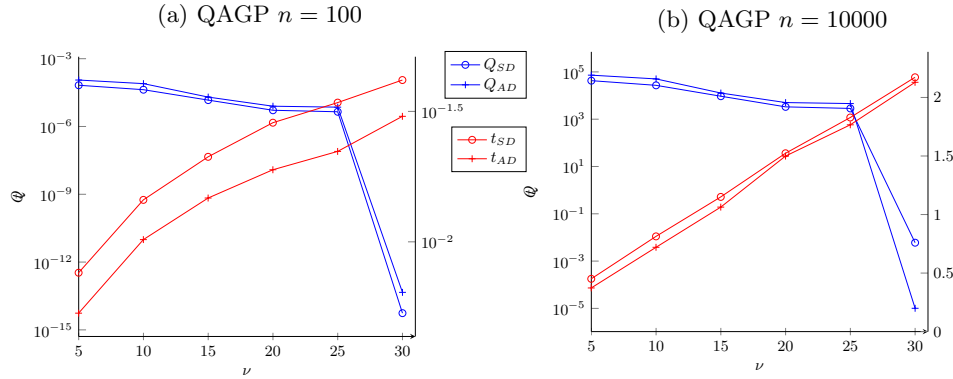


Fig. 10: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qagp routine using different approaches to differentiation.

10. **QNG nonadaptive Gauss-Kronrod integration:** The QNG algorithm is a non-adaptive procedure which uses fixed Gauss-Kronrod-Patterson abscissae to sample the integrand at a maximum of 87 points. It is provided for fast integration of smooth functions.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \sum_{i=1}^{n-1} x^{(\alpha_i + \alpha_{i+1})} \cdot \sin\left(\frac{\alpha_i}{x}\right)$$

where  $a(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i$  and  $b(\boldsymbol{\alpha}) = 3 \cdot \sum_{i=1}^n \alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using qng routine. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 11 and Table 6 respectively.

In this test case, the absolute error as well as relative error is set to  $10^{-7}$ . As illustrated in Figure 11, for this function with this accuracy, algorithmic tangent has nearly the same behaviour as symbolic tangent, symbolic adjoint spends less time than algorithmic adjoint for large  $ns$ . This routine is not adaptive and therefore it has always the same number of iterations. Memory requirement of algorithmic adjoint is significantly higher than memory requirement of symbolic adjoint.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.003	0.0004	0.003	0.0004	0.002
100	0.2	0.002	0.16	0.002	0.1
1000	13.52	0.01	13.54	0.02	8.08
5000	339.82	0.08	336.3	0.1	205.49
10000	-	0.21	-	0.21	-
100000	-	1.61	-	1.79	-
1000000	-	15.46	-	17.91	-
5000000	-	78.43	-	90.26	-

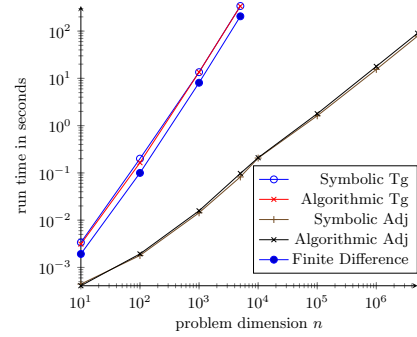


Fig. 11: Run time overhead in seconds for qng routine. Missing values indicate failure to converge within 500 seconds.

n	Symbolic	Algorithmic
	Adj	Adj
10	0.002	0.05
100	0.02	0.51
1000	0.22	5.05
5000	1.11	25.26
10000	2.21	50.51
100000	22.12	505.07
1000000	221.25	5050.68
5000000	1106.26	25253.4

Table 6: Memory Requirement in MB for qng routine.

11. **QAWF adaptive integration for Fourier integrals:** This function attempts to compute a Fourier integral of the function  $f$  over the semi-infinite interval  $[a, +\infty)$ . The subintervals and their results are stored in the memory provided by `workspace`. The integration over each subinterval uses the memory provided by `cycle_workspace` as workspace for the QAWO algorithm.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{\infty} \sum_{i=1}^n \frac{\alpha_i^2}{(\alpha_i^2 + 1) \cdot \sqrt{x + \alpha_i}}$$

where  $a(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using `qawf` routine. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\boldsymbol{\alpha}$ , the computational overhead as well as memory requirement are shown in Figure 12 and Table 7 respectively.

In this test case, the absolute error is set to  $10^{-8}$ . To reach this accuracy, e.g. for  $n = 5000$ , the number of iterations for symbolic and algorithmic is 9 and 11 respectively. As illustrated in Figure 12, for this function with this accuracy, symbolic tangent is faster than algorithmic tangent, algorithmic adjoint and symbolic adjoint have the same behaviour.

Figure 13 illustrates the convergence (blue lines) and run time (red lines) for computing adjoints with different number of iterations  $\nu$  using different differentiation methods in the integration of the reference problem with `qawf` routine for  $n = 100$  and  $n = 100000$ . For both cases the convergence of



n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.01	0.002	0.01	0.01	0.004
50	0.07	0.004	0.05	0.01	0.04
100	0.2	0.01	0.18	0.01	0.14
500	2.95	0.02	3.62	0.03	2.8
1000	11.31	0.05	13.42	0.05	10.34
5000	272.4	0.22	308.36	0.25	239.56
10000	-	0.45	-	0.5	-
50000	-	1.89	-	2.11	-
100000	-	3.82	-	4.18	-
500000	-	16.42	-	16.45	-
1000000	-	33.07	-	33.96	-

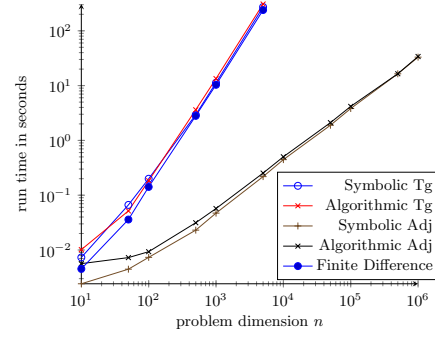


Fig. 12: Run time overhead in seconds for qawf routine. Missing values indicate failure to converge within 400 seconds.

n	Symbolic		Algorithmic	
	Adj	$\nu$	Adj	$\nu$
10	0.002	10	0.55	9
50	0.01	10	2.29	10
100	0.02	11	4.74	11
500	0.1	10	22.93	11
1000	0.2	10	42.41	12
5000	0.99	9	195.1	11
10000	1.98	9	390.03	11
50000	9.92	7	1625.2	9
100000	19.84	7	3250.27	9
500000	99.18	5	13008.2	7
1000000	198.36	5	26016.4	7

Table 7: Memory Requirement in MB and number of iterations ( $\nu$ ) for qawf routine.

symbolic is faster than algorithmic. Furthermore, symbolic adjoint takes less time than algorithmic adjoint for  $n = 100$ , however, it is not the case for  $n = 100000$ . This is because, qawf uses qawo in its implementation and in qawo table of Chebyshev moments should be computed.

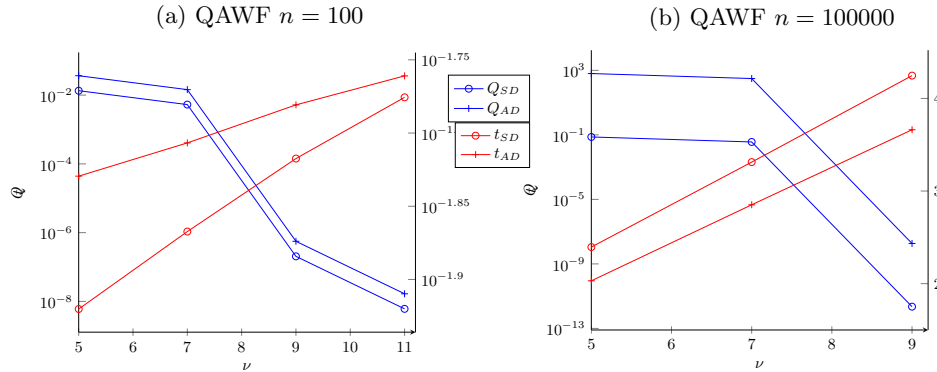


Fig. 13: Discrepancy and run time (in seconds) for evaluation of adjoints of the integration of the reference problem with qawf routine using different approaches to differentiation.

12. **GLFIXED Gauss-Legendre integration:** The fixed-order Gauss-Legendre integration routines are provided for fast integration of smooth functions with known polynomial order. The  $m$ -point Gauss-Legendre rule is exact for polynomials of order  $2 \cdot m - 1$  or less. Unlike other numerical integration routines within the library, these routines do not accept absolute or relative error bounds.

As case study, we consider evaluating the differentiation of the integral

$$I(\boldsymbol{\alpha}) = \int_{a(\boldsymbol{\alpha})}^{b(\boldsymbol{\alpha})} \sum_{i=1}^n \frac{20}{2 \cdot m - 1} \cdot \left( \left( \frac{3\alpha_i^2 x}{100} \right)^{2 \cdot m - 1} - \left( \frac{\alpha_i x}{10} \right)^{2 \cdot m - 1} \right)$$

where  $a(\boldsymbol{\alpha}) = \sum_{i=1}^n \alpha_i$  and  $b(\boldsymbol{\alpha}) = \sum_{i=1}^n 3\alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using glfixed routine. In this test case, we set  $m = 10$ . Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 14 and Table 8 respectively.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.001	0.0002	0.001	0.0001	0.0001
100	0.07	0.001	0.05	0.001	0.004
1000	3.68	0.01	3.62	0.005	0.34
5000	88.95	0.03	92.16	0.03	9.31
10000	360.91	0.05	371.51	0.05	34.31
100000	-	0.65	-	0.54	-
1000000	-	5.59	-	5.31	-
5000000	-	27.95	-	26.93	-

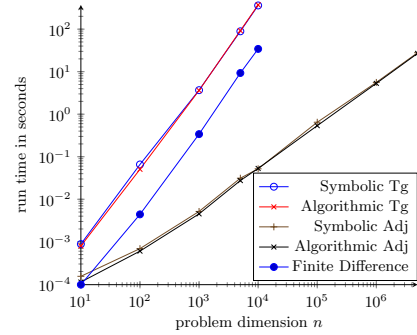


Fig. 14: Run time overhead in seconds for glfixed routine. Missing values indicate failure to converge within 500 seconds.

n	Symbolic	Algorithmic
	Adj	Adj
10	0.003	0.02
100	0.03	0.2
1000	0.3	2.04
5000	1.49	10.19
10000	2.98	20.37
100000	29.75	203.71
1000000	297.55	2037.05
5000000	1487.73	10185.3

Table 8: Memory Requirement in MB for glfixed routine.

As illustrated in Figure 14, for this function with this accuracy, symbolic tangent is faster than algorithmic tangent for large  $ns$ , algorithmic and symbolic adjoint have the same behaviour in terms of runtime, however, algorithmic adjoint requires higher memory requirement.

13. **CQUAD doubly-adaptive integration:** CQUAD is a new doubly-adaptive general-purpose quadrature routine which can handle most types of singularities, non-numerical function values such as *Inf* or *NaN*, as well as some divergent integrals. It generally requires more function evaluations than the integration routines in QUADPACK, yet fails less often for difficult integrands. The underlying algorithm uses a doubly-adaptive scheme in which Clenshaw-Curtis quadrature rules of increasing degree are used to compute the integral in each interval. The  $L_2$ -norm of the difference between the underlying interpolatory polynomials of two successive rules is used as an error estimate. The interval is subdivided if the difference between two successive rules is too large or a rule of maximum degree has been reached.

As case study, we consider evaluating the differentiation of the integral

$$I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} \sum_{i=1}^n \frac{\alpha_i^2}{\sqrt{(\alpha_i^2 + 1) \cdot x}}$$

where  $a(\alpha) = \sum_{i=1}^n \alpha_i$  and  $b(\alpha) = \sum_{i=1}^n 3\alpha_i^2$  with respect to its parameters  $\alpha_i > 0, i = 1, \dots, n$  using cquad routine. Differentiating this integral with algorithmic and symbolic tangent and adjoint for different dimensions of  $\alpha$ , the computational overhead as well as memory requirement are shown in Figure 15 and Table 9 respectively.

n	Symbolic		Algorithmic		Finite Diff.
	Tg	Adj	Tg	Adj	
10	0.0005	0.0002	0.0004	0.0002	0.0004
100	0.03	0.001	0.02	0.001	0.02
1000	1.51	0.007	1.34	0.003	1.06
5000	36.44	0.04	33.48	0.02	26.67
10000	157.85	0.07	134.93	0.05	107.57
100000	-	0.89	-	0.48	-
1000000	-	8.96	-	4.8	-

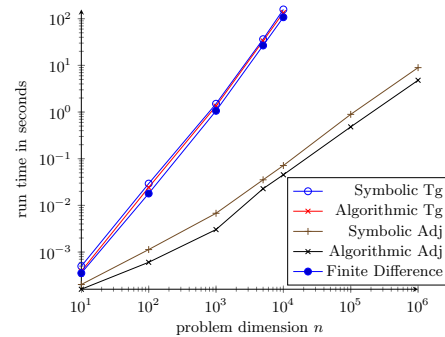


Fig. 15: Run time overhead in seconds for cquad routine. Missing values indicate failure to converge within 300 seconds.

n	Symbolic	Algorithmic
	Adj	Adj
10	0.002	0.12
100	0.02	0.47
1000	0.24	3.99
5000	1.18	19.61
10000	2.37	39.14
100000	23.65	390.71
1000000	236.51	3906.34

Table 9: Memory Requirement in MB for cquad routine.

In this test case, the absolute and relative error is set to  $10^{-12}$ . In the implementation of `cquad` routine with symbolic adjoint, additional operations should be done, in order to compute the  $n$  results and errors simultaneously. As illustrated in Figure 15, for this function with this accuracy, algorithmic tangent is faster than symbolic tangent, algorithmic adjoint has less runtime in comparison to other methods, however, algorithmic adjoint requires higher memory requirement.

## 6 Summary

In this paper we discussed algorithmic and symbolic differentiation of integrals with multi-dimensional parameters. The run time and memory overhead for algorithmic and symbolic approaches to the differentiation of the integrals with  $\nu_1$  and  $\nu_2$  (e.g. `qags`) iterations is shown in Table 10.

	Symbolic		Algorithmic	
	Tangent	Adjoint	Tangent	Adjoint
Memory	$O(n)$	$O(n)$	$O(n)$	$\nu_1 \cdot O(n)$
Run Time	$\nu_2 \cdot O(n)$	$\nu_2 \cdot O(1)$	$\nu_1 \cdot O(n)$	$\nu_1 \cdot O(1)$

Table 10: Computational complexity and memory requirement of  $n$  projections of the integral with algorithmic/symbolic tangent and adjoint modes of differentiation for  $\nu_1$  algorithmic and  $\nu_2$  symbolic (e.g. `qags`) iterations applied to the integrand/differentiation of the integrand with  $n$  parameters.

Computing the differentiation of the integral with symbolic tangent and algorithmic modes, the integration routine stays the same and just the data types should be changed. As shown in Section 1, in symbolic tangent/adjoint mode, the differentiation of the function should be passed to the integration routine. Evaluating the derivative of the integral with symbolic adjoint, the differentiation of the function is not scalar any more, but a vector. This should be considered in every function and routine of the integration and this is the reason to build vectorized functions and integration routines in `gsl` in order to make the results and errors be evaluated simultaneously.

In Section 5, we observe the differences between algorithmic and symbolic in evaluation of the derivative of the integrals with different routines. Note that, the number of iterations in symbolic and algorithmic is not always the same, because the algorithmic one integrates the function, whereas the symbolic one integrates the differentiation of the function with respect to parameters. Furthermore, in the symbolic tangent version, the numbers of iterations are different (or at least should not be the same) in every projection, due to integrating the differentiation of the function and having different values for each parameter, however, in algorithmic tangent all of the projections are done with the same number of iterations. Evaluation of tangents of the integrals with symbolic and algorithmic modes has nearly the same runtime overhead. This is also the case in evaluation of adjoints. Applying adjoint differentiation of the integrals is better alternative

than applying the tangent one, because of independence of the computational cost to the  $n$  in adjoint mode.

It is also shown that the runtime overhead of symbolic and algorithmic modes depends on the problem size  $n$  and number of iterations  $\nu$ . In computation of the adjoints, for small  $n$  and  $\nu$  algorithmic version is slightly faster, because of additional computation of  $f(\boldsymbol{\alpha}, b(\boldsymbol{\alpha}))\nabla b - f(\boldsymbol{\alpha}, a(\boldsymbol{\alpha}))\nabla a$  in the symbolic version. By increasing  $\nu$ , the symbolic mode will be faster, because the algorithmic one should go through the algorithm line by line  $\nu$  times and register the active variables for reverse interpretation and compute the derivatives. This requires memory as well as runtime. Increasing  $n$  and having the same  $\nu$ , symbolic and algorithmic would have the same runtime (Figure 10), except the cases that the table of Chebyshev moments should be computed, in this case because of increasing the dimensions Cheb12 and Cheb24 in every iteration with factor of  $n$  in vectorized gsl, the symbolic version requires more runtime (Figure 13).

Furthermore, in most of the integration routines the convergence of symbolic is faster than algorithmic one. Additionally, the memory requirement of algorithmic adjoint is significantly higher than the memory requirement of the symbolic adjoint.

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