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Abstract. In this paper we study the complexity of computing a behavioural pseudometric on Segala's probabilistic automata. The pseudometric concerned here stems from a real-valued logic (by Desharnais *et al*) and does not discount the future. We show that the problem whether the pseudometric between two states of a finite probabilistic automata is under a given threshold lies in the intersection of NP and coNP. This result significantly improves the previous PSPACE upperbound (by e.g. van Breugel *et al*).

1 Introduction

In recent years, probabilistic behavioural equivalences have been extensively studied. Many equivalences such as probabilistic bisimulation [LS91,SL95] and probabilistic simulation [JL91,SL95] have been established. And many efficient algorithms have been proposed for these equivalences [BEMC00,CS02]. Here we focus on probabilistic bisimulation. Generally, probabilistic bisimulation is a class of formal notions judging whether two probabilistic systems are behaviourally "equivalent". In practical problems, they are often used to compare an implementation of a system with the specification of the system, where both the implementation and the specification are modeled by formal probabilistic systems. Furthermore, one can also tackle the state explosion problem by reducing a large probabilistic system to its quotient system w.r.t probabilistic bisimulation. This is because the quotient system is often PCTL- or PCTL*-equivalent to the original one [ASB95,SL95].

However, the notion of probabilistic bisimulation relies on exact probability values and a slight variation of probability values will differentiate two originally equivalent probabilistic systems. In this sense, this notion is not robust. This is drastically the case when a probabilistic system contains physical objects (such as a wire) whose probability values are based on experimental results. In this case, the probabilistic values should not be treated as "exact values". Even in pure theoretical probabilistic models where probability values are indeed exact, an implementation of the model may still contain probability values which deviate slightly from their original counterparts. Thus, a notion of approximate bisimulation is really needed.

The notion of approximate bisimulation is first considered by Giacalone *et al* [GcJS90]. They defined a pseudometric over the states of a probabilistic transition system. This yields a smooth, quantitative notion of behavioural equivalence. A pseudometric differs from an ordinary metric in that different elements (i.e., states) can have distance 0. The distance between states, a real number between 0 and 1, can be used to express the similarity of the behaviour of the system started in those states. The smaller the distance, the more alike the behaviour is. In particular, the distance between states is 0 if they are indistinguishable.

Then the study of approximate bisimulation is divided into two directions. One direction is through a study of real-valued logic where the main work is by Josée Desharnais *et al* [DGJP04]. The other is through a study of coalgebra representation of probabilistic transition systems where the main work is by Franck van Breugel *et al* [vBW01b]. They both defined a pseudometric for probabilistic systems where states with zero distance are probabilistic bisimilar [LS91]. Both of the two pseudometrics are classified into future-discounting (*abbr.* FD) and non-future-discounting (*abbr.* nFD) version. In the FD version, differences that appear in the future are discounted and contribute less to the value of the pseudometric. While in nFD version, differences that appear in the future contribute fully to the pseudometric. NFD pseudometrics seems to be a more natural extension of probabilistic bisimulation, as in [Pan09,vBSW07] it is shown that the nFD pseudometric [DGJP04] defined through a real-valued logic can be defined in a style similar to probabilistic bisimulation [LS91]. Also the notion of nFD pseudometrics is important to analyze systems that are expected to run forever. For FD version, efficient approximation algorithms have been obtained on finite-state systems [vBW01a]. However for nFD version, the best-known complexity upperbound (before this paper) is only PSPACE [vBSW07,CHL09].

In this paper, we consider the nFD pseudometric over finite probabilistic automata [SL95] which is an extension of the nFD ones defined in [Pan09,vBSW07]. We show that the problem whether the pseudometric between two given states is under a given threshold lies in $\mathsf{NP} \cap \mathsf{coNP}$, which significantly improves previous PSPACE upperbound [vBSW07,CHL09].

2 Preliminaries

2.1 Finite Probabilistic Automata

In this subsection we define the notion of "finite probabilistic automata", which corresponds exactly to Segala's "simple probabilistic automata" [SL95] in the finite setting.

Definition 1 (Probability Distribution). *Let S be a countable set. A function $\mu : S \to [0,1]$ is a* probability distribution *over S if $\sum_{s\in S} \mu(s) = 1$ and $\mu(s) \in \mathbb{Q}$ for all $s \in S$. We denote the set of probability distributions over S by $\mathcal{D}(S)$.*

Definition 2 (Finite Probabilistic Automata). *A* finite probabilistic automata *(finite PA) $\mathcal{T}$ is a tuple (S, Act, Ω) where*

- *S is a finite set of* states*;*
- *Act is a finite set of* actions*;*
- *$\Omega \subseteq S \times Act \times \mathcal{D}(S)$ is a finite set of* transitions*.*

We define $\mathrm{Act}^{\mathcal{T}}(s) := \{a \in Act \mid \exists \mu.(s, a, \mu) \in \Omega\}$ *for each $s \in S$. We write $s \xrightarrow{a} \mu \in \Omega$ instead of $(s, a, \mu) \in \Omega$. We omit '$\mathcal{T}$' or 'Ω' in the notations above if the context is clear.*

Finite probabilistic automata extend Markov Chains studied in [vBSW07,Pan09] in the sense that a Labeled Markov Chain can be deemed as a finite probabilistic automata where the transition relation Ω is a function from $S \times Act$ to $\mathcal{D}(S)$.

2.2 Polyhedra and Linear Programming

We introduce polyhedra and linear programming along [Sch10]. Given a real-valued vector b, we denote b^i to be the i-th entry of b.

Definition 3 (Polyhedra). *A subset $H \subseteq \mathbb{R}^n$ is a* halfspace *iff there is $h \in \mathbb{R}^n$ and $e \in \mathbb{R}$ such that $H = \{x \in \mathbb{R}^n \mid h^T x \leq e\}$. A subset $P \subseteq \mathbb{R}^n$ is a* polyhedron *iff it is the intersection of a finite number of halfspaces. Equivalently $P \subseteq \mathbb{R}^n$ is a polyhedron iff there is an $m \times n$ matrix A and a vector $b \in \mathbb{R}^m$ such that $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$, where "$Ax \leq b$" is an abbreviation for "$(Ax)^i \leq b^i$ for all $1 \leq i \leq m$".*

Remark 1. Informally, a polyhedron is a finite intersection of constraints of the form $h^T x \leq e$. Note that constraints such as $h^T x \geq e$ or $h^T x = e$ are covered, since "$h^T x \geq e$" can be rewritten as "$(-h)^T x \leq -e$" and "$h^T x = e$" is equivalent to "$h^T x \leq e$ and $h^T x \geq e$".

An important concept to characterize polyhedra is the notion of vertices.

Definition 4 (Vertex). *Let P be a polyhedron. A point $z \in P$ is a* vertex *iff there does not exist points $x, y \in P$ and $\lambda \in [0, 1]$ such that $x \neq z$, $y \neq z$ and $z = \lambda \cdot x + (1 - \lambda) \cdot y$.*

The following theorem states that vertices of a polyhedron can be characterized by full-rank rows of the specifying matrix, whose proof can be found in [Sch10, Theorem 2.2].

Theorem 1. *Let $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ be a polyhedron in $\mathbb{R}^n$ and $z \in P$. Define the matrix A_z to be the submatrix of A consisting of all the row vectors h_i of A for which $h_i z = b^i$. Then z is a vertex of P iff $\mathrm{rank}(A_z) = n$.*

Definition 5 (Linear Programming). *A* linear programming problem *(LP) is specified by its type as well as an $m \times n$ matrix A and two vectors $b \in \mathbb{R}^m, c \in \mathbb{R}^n$. There are two types of LP's: MAX-LP and MIN-LP, whose forms are as follows:*

MAX-LP	*MIN-LP*
maximize $c^T x$ *subject to $Ax \leq b$*	*minimize $c^T x$* *subject to $Ax \leq b$*

Let Γ be an LP (either a MAX-LP or a MIN-LP) as above. The aim is to compute the optimal value *(denoted $\mathrm{OpV}(\Gamma)$) of Γ, which is defined as the maximum (for MAX-LP) or minimum value (for MIN-LP) of the set $\{c^T x \mid x \in \mathbb{R}^n$ and $Ax \leq b\}$. The set of* feasible solutions *of Γ, denoted $\mathrm{Poly}(\Gamma)$, is defined as the polyhedron $\{x \in \mathbb{R}^n \mid Ax \leq b\}$. The set of* optimal solutions *of Γ, denoted $\mathrm{OpS}(\Gamma)$, is defined as $\{x \in \mathrm{Poly}(\Gamma) \mid c^T x = \mathrm{OpV}(\Gamma)\}$.*

Note that the maximum or minimum value can be reached if the supremum or infimum is finite, since $\mathrm{Poly}(\Gamma)$ is a closed set and $x \mapsto c^T x$ is a continuous function.

The following theorem clarifies the relationship between vertices of polyhedra and optimal values of linear programming problems.

Theorem 2. *For any* LP *Γ, if $\mathrm{Poly}(\Gamma)$ is bounded, then the optimal value of Γ is reached at some vertex of $\mathrm{Poly}(\Gamma)$.*

Proof. See Theorem 2.3, Theorem 2.4 and Exercise 2.21 in [Sch10].

2.3 Distance Functions and Pseudometrics

In this subsection we introduce distance functions and pseudometrics. Below we fix a set $\mathcal{U}$.

Definition 6 (Distance Functions, Pseudometrics). *A function $d : \mathcal{U} \times \mathcal{U} \to [0,1]$ is a* distance function *over $\mathcal{U}$ iff $d(u,u) = 0$ for all $u \in \mathcal{U}$. A distance function d is further a* pseudometric *over $\mathcal{U}$ iff for all $u, v, w \in \mathcal{U}$, $d(u,v) = d(v,u)$ and $d(u,w) \leq d(u,v) + d(v,w)$. We denote the set of distance functions (*resp. *pseudometrics) over $\mathcal{U}$ by $\mathcal{M}_{\mathrm{d}}(\mathcal{U})$ (*resp. *$\mathcal{M}_{\mathrm{p}}(\mathcal{U})$).*

In the following we use 'κ' to indicate either 'd' or 'p'. In this way we can use "$\mathcal{M}_\kappa(\mathcal{U})$" to make arguments shared by both $\mathcal{M}_{\mathrm{d}}(\mathcal{U})$ and $\mathcal{M}_{\mathrm{p}}(\mathcal{U})$.

Definition 7 (The Partial Order). *The binary relation $\preceq_\kappa$ over $\mathcal{M}_\kappa(\mathcal{U})$ is defined as follows: given $d_1, d_2 \in \mathcal{M}_\kappa(\mathcal{U})$, $d_1 \preceq_\kappa d_2$ iff $d_1(u,v) \geq d_2(u,v)$ for all $u, v \in \mathcal{U}$.*

Clearly, $\preceq_\kappa$ is a partial order over $\mathcal{M}_\kappa(\mathcal{U})$.

Theorem 3. *$(\mathcal{M}_\kappa(\mathcal{U}), \preceq_\kappa)$ is a complete lattice.*

Proof. The bottom element $\bot_\kappa$ is determined by: $\bot_\kappa(u,u) = 0$ and $\bot_\kappa(u,v) = 1$ if $u \neq v$; for all $u, v \in \mathcal{U}$. The top element $\top_\kappa$ is defined by: $\top_\kappa(u,v) = 0$ for all $u, v \in \mathcal{U}$. Given $\mathcal{M} \subseteq \mathcal{M}_\kappa(\mathcal{U})$, the greatest lower bound $\bigsqcap \mathcal{M}$ is given by:

$$\left(\bigsqcap \mathcal{M}\right)(u,v) := \sup\{d(u,v) \mid d \in \mathcal{M}\} \text{ for arbitrary } u, v \in \mathcal{U};$$

And the least upper bound $\bigsqcup \mathcal{M}$ is given by: $\bigsqcup \mathcal{M} = \bigsqcap \{d \in \mathcal{M}_\kappa(\mathcal{U}) \mid \forall d' \in \mathcal{M}. d' \preceq_\kappa d\}$.

3 Distance Functions and Pseudometrics on Probabilistic Automata

In this section we define distance functions and pseudometrics on finite PA. Our pseudometric extends the one on Markov Chains [Pan09,vBSW07], which is defined as a maximum fixed point similar to probabilistic bisimulation [LS91] and shown to be equivalent to the non-future-discounting pseudometric derived from a real-valued logic [Pan09,vBSW07]. Below we fix a finite PA $\mathcal{T} = (S, Act, \Omega)$. We use κ to indicate either 'd' or 'p'.

Definition 8. *Let $d \in \mathcal{M}_\kappa(S)$. Then the element $\Delta(d) \in \mathcal{M}_\kappa(\mathcal{D}(S))$ is given by:*

$$\Delta(d)(\mu,\nu) := \mathrm{OpV}(\Delta_d^{\mu,\nu}), \text{ for all } \mu, \nu \in \mathcal{D}(S)$$

where the LP $\Delta_d^{\mu,\nu}$ is depicted in Tab. 1.

Intuitively, Definition 8 lifts an element of $\mathcal{M}_\kappa(S)$ to an element of $\mathcal{M}_\kappa(\mathcal{D}(S))$. This is however not the original lifting definition. The original one which is derived from the Kantorovich metric [Kan42] is captured by the following definition.

Definition 9. *Given a pseudometric $d \in \mathcal{M}_{\mathrm{p}}(S)$, the pseudometric $\Theta(d) \in \mathcal{M}_{\mathrm{p}}(\mathcal{D}(S))$ is given by: $\Theta(d)(\mu,\nu) := \mathrm{OpV}(\Theta_d^{\mu,\nu})$ for all $\mu, \nu \in \mathcal{D}(S)$, where $\Theta_d^{\mu,\nu}$ is depicted in Tab. 1.*

Table 1. Two Lifting Definitions from $\mathcal{M}_\kappa(S)$ to $\mathcal{M}_\kappa(\mathcal{D}(S))$

$\Theta_d^{\mu,\nu}$	$\Delta_d^{\mu,\nu}$
maximize $\sum_{s\in S}(\mu(s)-\nu(s))\cdot x_s$ subject to $\bullet\forall s\in S: 0\le x_s\le 1$ $\bullet\forall s,t\in S: x_s - x_t \le d(s,t)$	minimize $\sum_{s,t\in S} d(s,t)\cdot y_{s,t}$ subject to $\bullet\forall s\in S: \sum_{t\in S} y_{s,t}=\mu(s)$ $\bullet\forall t\in S: \sum_{s\in S} y_{s,t}=\nu(t)$ $\bullet\forall s,t\in S: y_{s,t}\ge 0$

By applying the Kantorovich-Rubinstein Duality Theorem [KR58] (cf. [vBSW07]) or the Duality Theorem of linear programming (cf. [vBW01a]), one can show that the two definitions meet on the pseudometrics over S.

Theorem 4. *For any $d \in \mathcal{M}_{\mathrm{p}}(S)$, $\Delta(d) = \Theta(d)$.*

The following two lemmas deal with some basic properties related to Definition 8.

Lemma 1 (Monotonicity). *For any $d_1, d_2 \in \mathcal{M}_\kappa(S)$, if $d_1 \preceq_\kappa d_2$ then $\Delta(d_1) \preceq_\kappa \Delta(d_2)$.*

Proof. Fix arbitrary $\mu, \nu \in \mathcal{D}(S)$. Denote $P := \mathrm{Poly}(\Delta_{d_1}^{\mu,\nu}) \left(= \mathrm{Poly}(\Delta_{d_2}^{\mu,\nu})\right)$. By $d_1 \preceq_\kappa d_2$, we have $\sum_{s,t\in S} d_1(s,t)\cdot y_{s,t} \ge \sum_{s,t\in S} d_2(s,t)\cdot y_{s,t}$ for all $\{y_{s,t}\}_{s,t\in S} \in P$. It follows that $\Delta(d_1)(\mu,\nu) \ge \Delta(d_2)(\mu,\nu)$. Then the result follows.

Lemma 2 (Continuity). *Let $d \in \mathcal{M}_\kappa(S)$ and $\{d_i\}_{i\in\mathbb{N}}$ for which each $d_i \in \mathcal{M}_\kappa(S)$. If $\lim\limits_{i\to\infty} d_i(s,t) = d(s,t)$ for all $s,t \in S$, then $\lim\limits_{i\to\infty} \Delta(d_i)(\mu,\nu) = \Delta(d)(\mu,\nu)$ for all $\mu,\nu \in \mathcal{D}(S)$.*

Proof. Fix arbitrary $\mu, \nu \in \mathcal{D}(S)$. Denote $P := \mathrm{Poly}(\Delta_{\mathfrak{d}}^{\mu,\nu})$ where the choice of $\mathfrak{d} \in \mathcal{M}_\kappa(S)$ is irrelevant here. By Theorem 2, for all $\mathfrak{d} \in \mathcal{M}_\kappa(S)$,

$$\Delta(\mathfrak{d})(\mu,\nu) = \min\{\sum_{s,t\in S} \mathfrak{d}(s,t)\cdot y_{s,t} \mid \{y_{s,t}\}_{s,t\in S} \text{ is a vertex of } P\}$$

By Theorem 1, the number of vertices of a polyhedron is finite. Then the equation above tells us that $\Delta(\mathfrak{d})(\mu,\nu)$ is continuous on $\mathfrak{d}$. Thus we have $\lim\limits_{i\to\infty} \Delta(d_i)(\mu,\nu) = \Delta(d)(\mu,\nu)$.

Then we define a monotone function $\mathcal{F}_\kappa$ on $\mathcal{M}_\kappa(S)$ as follows.

Definition 10. *The function $\mathcal{F}_\kappa : \mathcal{M}_\kappa(S) \to \mathcal{M}_\kappa(S)$ is given by: for any $d \in \mathcal{M}_\kappa(S)$,*

$$\mathcal{F}_\kappa(d)(s,t) := \max_{a\in Act}\left\{\max\left(\sup_{s\xrightarrow{a}\mu}\inf_{t\xrightarrow{a}\nu}\Delta(d)(\mu,\nu), \sup_{t\xrightarrow{a}\nu}\inf_{s\xrightarrow{a}\mu}\Delta(d)(\mu,\nu)\right)\right\}$$

for arbitrary $s,t \in S$, where $\inf\emptyset := 1$ and $\sup\emptyset := 0$.

One can prove by definition that $\mathcal{F}_\kappa$ is indeed a function $\mathcal{M}_\kappa(S) \to \mathcal{M}_\kappa(S)$ (especially when $\kappa = \mathrm{p}$). Furthermore, it can be proved from Definition 10 and Lemma 1 that $\mathcal{F}_\kappa$ is indeed a monotone function.

Lemma 3. *$\mathcal{F}_\kappa$ is monotone on $\mathcal{M}_\kappa(S)$, i.e., $d_1 \preceq_\kappa d_2$ implies $\mathcal{F}_\kappa(d_1) \preceq_\kappa \mathcal{F}_\kappa(d_2)$.*

Remark 2. By definition, $\mathrm{Act}(s) \neq \mathrm{Act}(t)$ implies $\mathcal{F}_\kappa(\mathfrak{d})(s,t) = 1$ and $\mathrm{Act}(s) = \mathrm{Act}(t) = \emptyset$ implies $\mathcal{F}_\kappa(\mathfrak{d})(s,t) = 0$, regardless of the choice of $\mathfrak{d}$.

Since $\mathcal{F}_\kappa$ is monotone, it has a greatest fixed point. This is formulated as follows.

Definition 11. *Define $\mathbf{d}_\kappa \in \mathcal{M}_\kappa(S)$ to be the greatest fixed point of $\mathcal{F}_\kappa$. By Tarski Fixed-Point Theorem, $\mathbf{d}_\kappa = \bigsqcup\{d \in \mathcal{M}_\kappa(S) \mid d \preceq_\kappa \mathcal{F}_\kappa(d)\}$.*

The pseudometric $\boldsymbol{d}_{\mathrm{p}}$ is closely related with strong bisimulation [SL95] on probabilistic automata: s and t are bisimilar iff $\boldsymbol{d}_{\mathrm{p}}(s,t) = 0$. Here we skip this due to lack of space. One may see [Pan09,vBSW07] for more information. Below we define approximants $\{\boldsymbol{d}^i_\kappa\}_{i\in\mathbb{N}_0}$ of $\boldsymbol{d}_\kappa$.

Definition 12. *The family $\{\mathbf{d}^i_\kappa\}_{i\in\mathbb{N}_0}$ of approximants of $\mathbf{d}_\kappa$ is inductively defined as follows: $\mathbf{d}^0_\kappa := \top_\kappa$ (i.e., $\mathbf{d}^0_\kappa(s,t) = 0$ for all $s,t \in S$); $\mathbf{d}^{i+1}_\kappa = \mathcal{F}_\kappa(\mathbf{d}^i_\kappa)$.*

Since $\mathcal{F}_\kappa$ is a monotone function, the sequence $\{\boldsymbol{d}^i_\kappa\}_{i\in\mathbb{N}_0}$ is decreasing w.r.t $\preceq_\kappa$. The following theorem shows that $\boldsymbol{d}_\kappa$ is the limitation of the sequence.

Theorem 5. *For any $s,t \in S$, $\boldsymbol{d}_\kappa(s,t) = \lim\limits_{i\to\infty} \boldsymbol{d}^i_\kappa(s,t)$.*

Proof. Let $\boldsymbol{d}^\infty_\kappa \in \mathcal{M}_\kappa(S)$ be given by $\boldsymbol{d}^\infty_\kappa(s,t) := \lim\limits_{i\to\infty} \boldsymbol{d}^i_\kappa(s,t)$. We prove that $\boldsymbol{d}^\infty_\kappa = \boldsymbol{d}_\kappa$.

$\boldsymbol{d}_\kappa \preceq_\kappa \boldsymbol{d}^\infty_\kappa$: By $\boldsymbol{d}_\kappa \preceq_\kappa \boldsymbol{d}^0_\kappa$ and the monotonicity of $\mathcal{F}$, we can prove by induction on i that $\boldsymbol{d}_\kappa \preceq_\kappa \boldsymbol{d}^i_\kappa$ for all $i \in \mathbb{N}_0$. Thus $\boldsymbol{d}_\kappa \preceq_\kappa \boldsymbol{d}^\infty_\kappa$.

$\boldsymbol{d}^\infty_\kappa \preceq_\kappa \boldsymbol{d}_\kappa$: We prove that $\boldsymbol{d}^\infty_\kappa \preceq_\kappa \mathcal{F}_\kappa(\boldsymbol{d}^\infty_\kappa)$. Fix arbitrary $(s,t) \in S \times S$. Consider any $s \xrightarrow{a} \mu$. By Definition 12, for any $i \in \mathbb{N}$, there exists $t \xrightarrow{a} \nu_i$ such that $\Delta(\boldsymbol{d}^i_\kappa)(\mu,\nu_i) \leq \boldsymbol{d}^{i+1}_\kappa(s,t) \leq \boldsymbol{d}^\infty_\kappa(s,t)$. Since $\mathcal{T}$ is finite, there exists $t \xrightarrow{a} \nu$ such that $\Delta(\boldsymbol{d}^i_\kappa)(\mu,\nu) \leq \boldsymbol{d}^{i+1}_\kappa(s,t) \leq \boldsymbol{d}^\infty_\kappa(s,t)$ for infinitely many $i \in \mathbb{N}$. Then since $\{\boldsymbol{d}^i_\kappa\}_{i\in\mathbb{N}_0}$ is decreasing w.r.t $\preceq_\kappa$ and by Lemma 1, there exists $i \in \mathbb{N}$ such that for all $j \geq i$, $\Delta(\boldsymbol{d}^j_\kappa)(\mu,\nu) \leq \boldsymbol{d}^\infty_\kappa(s,t)$. Thus $\Delta(\boldsymbol{d}^\infty_\kappa)(\mu,\nu) \leq \boldsymbol{d}^\infty_\kappa(s,t)$ by Lemma 2. Similar arguments can be made for all $t \xrightarrow{a} \nu$. Thus $\boldsymbol{d}^\infty_\kappa \preceq_\kappa \mathcal{F}_\kappa(\boldsymbol{d}^\infty_\kappa)$.

Following Theorem 5 and the fact that $\boldsymbol{d}^0_{\mathrm{d}} = \boldsymbol{d}^0_{\mathrm{p}}$, we have the following corollary:

Corollary 1. $\boldsymbol{d}_{\mathrm{p}} = \boldsymbol{d}_{\mathrm{d}} = \bigsqcup\{d \in \mathcal{M}_{\mathrm{d}}(S) \mid d \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d)\}$

This corollary allows us to reason properties of $\boldsymbol{d}_{\mathrm{p}}$ on $\mathcal{M}_{\mathrm{d}}(S)$.

4 NP- and coNP-ness

In this section we study the complexity of the following problem:

- **Input:** a finite PA (S, Act, Ω), $s_{\mathrm{in}}, t_{\mathrm{in}} \in S$ and a number $\epsilon \in \mathbb{Q}_{\geq 0}$ represented in binary
- **Output:** whether $\boldsymbol{d}_{\mathrm{p}}(s_{\mathrm{in}}, t_{\mathrm{in}}) \leq \epsilon$ or not

More precisely, we consider the language

$$\mathcal{L} := \{\langle\langle S, Act, \Omega\rangle, \langle s_{\mathrm{in}}, t_{\mathrm{in}}\rangle, \epsilon\rangle \mid \boldsymbol{d}_{\mathrm{p}}(s_{\mathrm{in}}, t_{\mathrm{in}}) \leq \epsilon\}$$

We prove that $\mathcal{L} \in \mathsf{NP}\cap\mathsf{coNP}$. Our proof method is divided into three steps: First we establish a characterization of the greatest fixed point $\boldsymbol{d}_\mathrm{p}$ (or equivalently $\boldsymbol{d}_\mathrm{d}$ since $\boldsymbol{d}_\mathrm{p} = \boldsymbol{d}_\mathrm{d}$); Then based on such characterization, we show that whether a given $d \in \mathcal{M}_\mathrm{p}(S)$ equals $\boldsymbol{d}_\mathrm{p}$ is polynomial-time decidable; Finally, we complete the proof by showing how we can guess a pseudometric $d \in \mathcal{M}_\mathrm{p}(S)$ which is also a fixed point of $\mathcal{F}_\mathrm{p}$.

Below we fix a finite PA $(S, \mathit{Act}, \Omega)$. First we present the characterization of the greatest fixed point d_p. To this end we introduce the notion of "self-fulfilling" sets, as follows:

Definition 13. *Let $d \in \mathcal{M}_\mathrm{p}(S)$ with $d = \mathcal{F}_\mathrm{p}(d)$. A subset $X \subseteq S \times S$ is* self-fulfilling *w.r.t d iff for any $(s,t) \in X$, the following three conditions hold:*

1. *$d(s,t) > 0$ (which implies that $s \neq t$) and $\mathrm{Act}(s) = \mathrm{Act}(t)$;*
2. *for any $s \xrightarrow{a} \mu$ such that $d(s,t) = \inf\{\Delta(d)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$, there is $t \xrightarrow{a} \nu_0$ and $y = \{y_{s,t}\}_{s,t\in S} \in \mathrm{OpS}(\Delta_d^{\mu,\nu_0})$ such that $d(s,t) = \Delta(d)(\mu,\nu_0)$ and $\lfloor y \rfloor \subseteq X$.*
3. *for any $t \xrightarrow{a} \nu$ such that $d(s,t) = \inf\{\Delta(d)(\mu,\nu) \mid s \xrightarrow{a} \mu\}$, there is $s \xrightarrow{a} \mu_0$ and $y = \{y_{s,t}\}_{s,t\in S} \in \mathrm{OpS}(\Delta_d^{\mu_0,\nu})$ such that $d(s,t) = \Delta(d)(\mu_0,\nu)$ and $\lfloor y \rfloor \subseteq X$.*

where $\lfloor y \rfloor := \{(s,t) \in S \times S \mid y_{s,t} > 0\}$.

Intuitively, a self-fulfilling set X w.r.t d is a set such that all values $d(s,t)$ of d in X can be reached on X itself. Below we show that nonempty self-fulfilling sets characterize exactly the greatest fixed point $\boldsymbol{d}_\mathrm{p}$ of the function $\mathcal{F}_\mathrm{p}$.

Theorem 6. *Let $d \in \mathcal{M}_\mathrm{p}(S)$ such that $d = \mathcal{F}_\mathrm{p}(d)$. If $d \neq \boldsymbol{d}_\mathrm{p}$, then there exists a nonempty self-fulfilling set $X \subseteq S \times S$ with respect to d.*

Proof. Suppose $d \neq \boldsymbol{d}_\mathrm{p}$, we construct a nonempty self-fulfilling set X as described below. Denote $\delta(s,t) = d(s,t) - \boldsymbol{d}_\mathrm{p}(s,t)$. Then $\delta(s,t) \geq 0$ for all $s,t \in S$, and there is $(s,t) \in S \times S$ such that $\delta(s,t) > 0$ due to $d \neq \boldsymbol{d}_\mathrm{p}$. Define X to be the following set:

$$X := \{(s,t) \in S \times S \mid \delta(s,t) = \max\{\delta(s',t') \mid (s',t') \in S \times S\}\}$$

We prove that X is a nonempty self-fulfilling set. The non-emptiness of X is obvious. We further prove that any $(s,t) \in X$ satisfies the three conditions specified in Definition 13. Fix arbitrary $(s,t) \in X$. The analysis is as follows:

1. It is clear that $d(s,t) > 0$ since $\delta(s,t) > 0$. We prove that $\mathrm{Act}(s) = \mathrm{Act}(t)$. Suppose $\mathrm{Act}(s) \neq \mathrm{Act}(t)$. Then by Remark 2, $d(s,t) = \boldsymbol{d}_\mathrm{p}(s,t) = 1$ which implies $\delta(s,t) = 0$. Contradiction. So (s,t) satisfies the first condition in Definition 13.
2. Suppose that $s \xrightarrow{a} \mu$ satisfies $d(s,t) = \inf\{\Delta(d)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$ and $t \xrightarrow{a} \nu_0$ be arbitrarily chosen satisfying $\Delta(\boldsymbol{d}_\mathrm{p})(\mu,\nu_0) = \inf\{\Delta(\boldsymbol{d}_\mathrm{p})(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. Choose an arbitrary $y = \{y_{s,t}\}_{s,t\in S} \in \mathrm{OpS}(\Delta_{\boldsymbol{d}_\mathrm{p}}^{\mu,\nu_0})$. We prove that $y \in \mathrm{OpS}(\Delta_d^{\mu,\nu_0})$ and $\lfloor y \rfloor \subseteq X$ (cf. Definition 13). By the definition of X, we have $\delta(s',t') \leq \delta(s,t)$ for all $(s',t') \in S \times S$. Thus for any $\{y^*_{s,t}\}_{s,t\in S} \in \mathrm{Poly}(\Delta_d^{\mu,\nu_0})$, we have

$$\sum_{s',t'\in S} \boldsymbol{d}_\mathrm{p}(s',t') \cdot y^*_{s',t'} \geq \sum_{s',t'\in S} (d(s',t') - \delta(s,t)) \cdot y^*_{s',t'} \tag{1}$$

Since $\sum_{s',t'\in S} y^*_{s',t'} = 1$, we can further simplify Inequality (1) as follows:

$$\sum_{s',t'\in S} \boldsymbol{d}_{\mathrm{p}}(s',t')\cdot y^*_{s',t'} \geq \sum_{s',t'\in S} d(s',t')\cdot y^*_{s',t'} - \delta(s,t) \tag{2}$$

Note that $\mathrm{Poly}(\Delta^{\mu,\nu_0}_{d}) = \mathrm{Poly}(\Delta^{\mu,\nu_0}_{\boldsymbol{d}_{\mathrm{p}}})$. So by taking the infimum at the both sides of Inequality 2, we obtain that $\Delta(\boldsymbol{d}_{\mathrm{p}})(\mu,\nu_0) \geq \Delta(d)(\mu,\nu_0) - \delta(s,t)$. Further from Definition 10, we obtain $\boldsymbol{d}_{\mathrm{p}}(s,t) \geq \Delta(\boldsymbol{d}_{\mathrm{p}})(\mu,\nu_0)$. Thus, we have:

$$\boldsymbol{d}_{\mathrm{p}}(s,t) \geq \Delta(\boldsymbol{d}_{\mathrm{p}})(\mu,\nu_0) \geq \Delta(d)(\mu,\nu_0) - \delta(s,t) \geq d(s,t) - \delta(s,t) = \boldsymbol{d}_{\mathrm{p}}(s,t)$$

This means that $\boldsymbol{d}_{\mathrm{p}}(s,t) = \Delta(\boldsymbol{d}_{\mathrm{p}})(\mu,\nu_0)$ and $d(s,t) = \Delta(d)(\mu,\nu_0)$. Then we can form another inequality series as follows:

$$\begin{aligned}
&\textstyle\sum_{s',t'\in S} d(s',t')\cdot y_{s',t'} \\
&\geq d(s,t) && (\text{by } d(s,t) = \Delta(d)(\mu,\nu_0)) \\
&= \boldsymbol{d}_{\mathrm{p}}(s,t) + \delta(s,t) \\
&= \textstyle\sum_{s',t'\in S} \boldsymbol{d}_{\mathrm{p}}(s',t')\cdot y_{s',t'} + \delta(s,t) && (\text{by } \boldsymbol{d}_{\mathrm{p}}(s,t) = \Delta(\boldsymbol{d}_{\mathrm{p}})(\mu,\nu_0)) \\
&= \textstyle\sum_{s',t'\in S} (\boldsymbol{d}_{\mathrm{p}}(s',t') + \delta(s,t))\cdot y_{s',t'} \\
&\geq \textstyle\sum_{s',t'\in S} (\boldsymbol{d}_{\mathrm{p}}(s',t') + \delta(s',t'))\cdot y_{s',t'} \\
&= \textstyle\sum_{s',t'\in S} d(s',t')\cdot y_{s',t'}
\end{aligned}$$

Thus it must be the case that $y \in \mathrm{OpS}(\Delta^{\mu,\nu_0}_{d})$ and $\delta(s',t') = \delta(s,t)$ whenever $y_{s',t'} > 0$. Then we have $\lfloor y \rfloor \subseteq X$. So (s,t) satisfies the second condition in Definition 13.

3. Symmetrically we can prove that (s,t) satisfies the third condition in Definition 13.

Hence in conclusion, X is a self-fulfilling set.

Theorem 7. *Let $d \in \mathcal{M}_{\mathrm{p}}(S)$ such that $d = \mathcal{F}_{\mathrm{p}}(d)$. If there exists a nonempty self-fulfilling set $X \subseteq S \times S$ with respect to d, then $d \neq \boldsymbol{d}_{\mathrm{p}}$.*

Proof. Suppose $X \subseteq S \times S$ is a nonempty self-fulfilling set w.r.t d. We construct a distance function $d' \neq d$ such that $d' \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d')$ and $d \preceq_{\mathrm{d}} d'$. For all $(s \xrightarrow{a} \mu, t)$ and all $(s, t \xrightarrow{a} \nu)$ such that $(s,t) \in X$, we define the following differences: (note that $\mathrm{Act}(s) = \mathrm{Act}(t)$)

- $\delta[s \xrightarrow{a} \mu, t] := d(s,t) - \inf\{\Delta(d)(\mu,\nu') \mid t \xrightarrow{a} \nu'\}$;
- $\delta[s, t \xrightarrow{a} \nu] := d(s,t) - \inf\{\Delta(d)(\mu',\nu) \mid s \xrightarrow{a} \mu'\}$.

All the differences defined above are non-negative since $d = \mathcal{F}_{\mathrm{p}}(d)$. Further we define the following two differences: (where $\min\emptyset := 0$)

- $\delta_1 := \min\{\delta[s \xrightarrow{a} \mu, t] \mid (s,t) \in X, s \xrightarrow{a} \mu \text{ and } \delta[s \xrightarrow{a} \mu, t] > 0\}$;
- $\delta_2 := \min\{\delta[s, t \xrightarrow{a} \nu] \mid (s,t) \in X, t \xrightarrow{a} \nu \text{ and } \delta[s, t \xrightarrow{a} \nu] > 0\}$.

Finally we define the difference value δ as follows:

$$\delta := \begin{cases} \min\{\delta_1, \delta_2, \min\{d(s,t) \mid (s,t) \in X\}\} & \text{if } \delta_1 \neq 0 \text{ and } \delta_2 \neq 0 \\ \min\{\delta_2, \min\{d(s,t) \mid (s,t) \in X\}\} & \text{if } \delta_1 = 0 \text{ and } \delta_2 \neq 0 \\ \min\{\delta_1, \min\{d(s,t) \mid (s,t) \in X\}\} & \text{if } \delta_1 \neq 0 \text{ and } \delta_2 = 0 \\ \min\{d(s,t) \mid (s,t) \in X\} & \text{if } \delta_1 = 0 \text{ and } \delta_2 = 0 \end{cases}$$

Note that $\delta > 0$. Then we construct $d' \in \mathcal{M}_{\mathrm{d}}(S)$ by:

$$d'(s,t) := \begin{cases} d(s,t) - \frac{1}{2}\delta & \text{if } (s,t) \in X \\ d(s,t) & \text{if } (s,t) \notin X \end{cases}$$

It is clear that $d' \neq d$ since X is non-empty. We prove that $d' \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d')$. Fix an arbitrary $(s,t) \in S \times S$. Suppose $(s,t) \notin X$. Then from $d \preceq_{\mathrm{d}} d'$ we have $\mathcal{F}_{\mathrm{d}}(d) \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d')$. And then we have $d'(s,t) = d(s,t) = \mathcal{F}_{\mathrm{p}}(d)(s,t) = \mathcal{F}_{\mathrm{d}}(d)(s,t) \geq \mathcal{F}_{\mathrm{d}}(d')(s,t)$. Thus $d'(s,t) \geq \mathcal{F}_{\mathrm{d}}(d')(s,t)$. Suppose now that $(s,t) \in X$. For any $s \xrightarrow{a} \mu$, we clarify two cases below:

Case 1: $\delta[s \xrightarrow{a} \mu, t] > 0$. Then $\delta_1 > 0$. By definition, we have:

$$d'(s,t) \geq d(s,t) - \frac{1}{2}\delta > d(s,t) - \delta[s \xrightarrow{a} \mu, t] = \inf\{\Delta(d)(\mu,\nu') \mid t \xrightarrow{a} \nu'\}$$

From $d \preceq_{\mathrm{d}} d'$, we have $\inf\{\Delta(d)(\mu,\nu') \mid t \xrightarrow{a} \nu'\} \geq \inf\{\Delta(d')(\mu,\nu') \mid t \xrightarrow{a} \nu'\}$. Thus $d'(s,t) \geq \inf\{\Delta(d')(\mu,\nu') \mid t \xrightarrow{a} \nu'\}$.

Case 2: $\delta[s \xrightarrow{a} \mu, t] = 0$. Then since X is self-fulfilling there is $t \xrightarrow{a} \nu_0$ and $y = \{y_{s',t'}\}_{s',t' \in S} \in \mathrm{OpS}(\Delta_d^{\mu,\nu_0})$ such that $d(s,t) = \Delta(d)(\mu,\nu_0)$ and $\lfloor y \rfloor \subseteq X$. From $\lfloor y \rfloor \subseteq X$ we obtain

$$\sum_{s',t' \in S} d'(s',t') \cdot y_{s',t'} = \sum_{s',t' \in S} d(s',t') \cdot y_{s',t'} - \frac{1}{2}\delta = \Delta(d)(\mu,\nu_0) - \frac{1}{2}\delta = d'(s,t)$$

Then we have: $\inf\{\Delta(d')(\mu,\nu') \mid t \xrightarrow{a} \nu'\} \leq \Delta(d')(\mu,\nu_0) \leq \sum_{s',t' \in S} d'(s',t') \cdot y_{s',t'} = d'(s,t)$. Thus $d'(s,t) \geq \inf\{\Delta(d')(\mu,\nu') \mid t \xrightarrow{a} \nu'\}$ for all $s \xrightarrow{a} \mu$. Similarly we can prove that $d'(s,t) \geq \inf\{\Delta(d')(\mu',\nu) \mid s \xrightarrow{a} \mu'\}$ for all $t \xrightarrow{a} \nu$. Thus $d'(s,t) \geq \mathcal{F}_{\mathrm{d}}(d')(s,t)$ for all $(s,t) \in S \times S$. Hence $d' \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d')$. By $d \preceq_{\mathrm{d}} d'$ we have $d \neq \bigsqcup\{\mathfrak{d} \in \mathcal{M}_{\mathrm{d}}(S) | \mathfrak{d} \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(\mathfrak{d})\}$. Then by Corollary 1, we obtain that $d \neq \boldsymbol{d}_{\mathrm{p}}$.

Thus for any $d \in \mathcal{M}_{\mathrm{p}}(S)$ satisfying $d = \mathcal{F}_{\mathrm{p}}(d)$, $d \neq \boldsymbol{d}_{\mathrm{p}}$ iff there exists a nonempty self-fulfiling set w.r.t d. This characterization means that to check whether $d \neq \boldsymbol{d}_{\mathrm{p}}$ or not, we can equivalently check whether there exists a nonempty self-fulfilling set or not. The intuition here is that for any self-fulfilling sets X, Y, $X \cup Y$ is still a self-fulfilling set; thus there exists a largest self-fulfilling set. This gives rise to a refinement algorithm that computes the largest self-fulfilling set.

Theorem 8. *Denote* $\mathrm{FP} := \{d \in \mathcal{M}_{\mathrm{p}}(S) \mid d = \mathcal{F}_{\mathrm{p}}(d)\}$ *to be the set of fixed points of* $\mathcal{F}_{\mathrm{p}}$*. The problem whether a given* $d \in \mathrm{FP}$ *equals* $\boldsymbol{d}_{\mathrm{p}}$ *is decidable in polynomial time.*

Proof. From Theorem 6 and Theorem 7, we can solve the problem by checking whether there exists a nonempty self-fulfilling set w.r.t the given $d \in \mathrm{FP}$. Note that for any self-fulfilling sets X, Y w.r.t d, $X \cup Y$ is still self-fulfilling w.r.t d. So there exists a largest self-fulfilling set w.r.t d, which is denoted as Z. Then there exists a nonempty self-fulfilling set w.r.t d iff Z is nonempty. Below we develop a refinement algorithm to compute Z.

First we define a refinement function $ref : \mathcal{E} \to \mathcal{E}$, where the set $\mathcal{E}$ is given as follows:

$$\mathcal{E} := \{X \subseteq S \times S \mid \mathrm{Act}(s) = \mathrm{Act}(t) \text{ and } d(s,t) > 0 \text{ for all } (s,t) \in X\}$$

Note that $\mathcal{E}$ is nonempty since $\emptyset \in \mathcal{E}$. Given $X \in \mathcal{E}$, we define $\delta_X := \min\{d(s,t) \mid (s,t) \in X\}$ (where $\min \emptyset = 0$) and the distance function $d_X \in \mathcal{M}_{\mathrm{d}}(S)$ as follows:

$$d_X(s,t) = \begin{cases} d(s,t) - \delta_X & \text{if } (s,t) \in X \\ d(s,t) & \text{if } (s,t) \notin X \end{cases}$$

Then the set $ref(X) \in \mathcal{E}$ is defined as follows: $(s,t) \in ref(X)$ iff $(s,t) \in X$ and further (s,t) satisfies the following two conditions:

1. for all $s \xrightarrow{a} \mu$, if $d(s,t) = \inf\{\mathit{\Delta}(d)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$ then:
$$d_X(s,t) \geq \inf\{\mathit{\Delta}(d_X)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$$
2. for all $t \xrightarrow{a} \nu$, if $d(s,t) = \inf\{\mathit{\Delta}(d)(\mu,\nu) \mid s \xrightarrow{a} \mu\}$ then:
$$d_X(s,t) \geq \inf\{\mathit{\Delta}(d_X)(\mu,\nu) \mid s \xrightarrow{a} \mu\}$$

Note that $ref(X) \subseteq X$ for all $X \in \mathcal{E}$. Now we construct a sequence $\{Z_i\}_{i \in \mathbb{N}_0}$ as follows:

$$Z_0 = \{(s,t) \in S \times S \mid \mathrm{Act}(s) = \mathrm{Act}(t) \text{ and } d(s,t) > 0\};\ Z_{i+1} = ref(Z_i)$$

By $Z_{i+1} \subseteq Z_i$, there exists $n \leq |Z_0|$ such that $Z_{n+1} = ref(Z_n) = Z_n$. We show that $Z_n = Z$.

"$Z \subseteq Z_n$": We prove by induction that $Z \subseteq Z_i$ for all $i \in \mathbb{N}_0$. The base step $Z \subseteq Z_0$ is clear from the definition. For inductive step, suppose that $Z \subseteq Z_i$. We show that $Z \subseteq Z_{i+1} (= ref(Z_i))$. Fix arbitrary $(s,t) \in Z$. Consider any $s \xrightarrow{a} \mu$ such that $d(s,t) = \inf\{\mathit{\Delta}(d)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. Since Z is self-fulfilling, there is $t \xrightarrow{a} \nu_0$ and $y = \{y_{s',t'}\}_{s',t' \in S} \in \mathrm{OpS}(\mathit{\Delta}_d^{\mu,\nu_0})$ such that $d(s,t) = \mathit{\Delta}(d)(\mu,\nu_0)$ and $\lfloor y \rfloor \subseteq Z$. Since $Z \subseteq Z_i$, we have $d_{Z_i}(s,t) = d(s,t) - \delta_{Z_i}$ and $d_{Z_i}(s',t') = d(s',t') - \delta_{Z_i}$ for all $(s',t') \in \lfloor y \rfloor$. Thus we obtain

$$\sum_{s',t' \in S} d_{Z_i}(s',t') \cdot y_{s',t'} = \sum_{s',t' \in S} (d(s',t') - \delta_{Z_i}) \cdot y_{s',t'} = d(s,t) - \delta_{Z_i} = d_{Z_i}(s,t)$$

Hence $d_{Z_i}(s,t) \geq \mathit{\Delta}(d_{Z_i})(\mu,\nu_0) \geq \inf\{\mathit{\Delta}(d_{Z_i})(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. By a similar reasoning we can prove that for all $t \xrightarrow{a} \nu$, if $d(s,t) = \inf\{\mathit{\Delta}(d)(\mu,\nu) \mid s \xrightarrow{a} \mu\}$ then $d_{Z_i}(s,t) \geq \inf\{\mathit{\Delta}(d_{Z_i})(\mu,\nu) \mid s \xrightarrow{a} \mu\}$. So $(s,t) \in Z_{i+1}$. Thus $Z \subseteq Z_{i+1}$.

"$Z_n \subseteq Z$": We prove that Z_n is a self-fulfilling set, i.e., Z_n satisfies the three conditions specified in Definition 13. W.l.o.g we can assume that $Z_n \neq \emptyset$. The first condition in Definition 13 is directly satisfied since $Z_n \subseteq Z_0$. As for the second condition, consider any $(s,t) \in Z_n$ and $s \xrightarrow{a} \mu$ satisfying $d(s,t) = \inf\{\mathit{\Delta}(d)(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. By $Z_n = ref(Z_n)$, $d_{Z_n}(s,t) \geq \inf\{\mathit{\Delta}(d_{Z_n})(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. Choose $t \xrightarrow{a} \nu_0$ such that $\mathit{\Delta}(d_{Z_n})(\mu,\nu_0) = \inf\{\mathit{\Delta}(d_{Z_n})(\mu,\nu) \mid t \xrightarrow{a} \nu\}$ and an arbitrary $y \in \mathrm{OpS}(\mathit{\Delta}_{d_{Z_n}}^{\mu,\nu_0})$. Then since $d_{Z_n}(s,t) = d(s,t) - \delta_{Z_n}$, we have

$$\begin{aligned} d(s,t) &\geq \mathit{\Delta}(d_{Z_n})(\mu,\nu_0) + \delta_{Z_n} \\ &= \textstyle\sum_{s',t' \in S} (d_{Z_n}(s',t') + \delta_{Z_n}) \cdot y_{s',t'} \\ &\geq \textstyle\sum_{s',t' \in S} d(s',t') \cdot y_{s',t'} \\ &\geq \mathit{\Delta}(d)(\mu,\nu_0) \\ &\geq d(s,t) \end{aligned}$$

Thus $d(s,t) = \mathit{\Delta}(d)(\mu,\nu_0)$, $y \in \mathrm{OpS}(\mathit{\Delta}_d^{\mu,\nu_0})$ and $d_{Z_n}(s',t') = d(s',t') - \delta_{Z_n}$ for all $(s',t') \in \lfloor y \rfloor$, which implies that $\lfloor y \rfloor \subseteq Z_n$ since $\delta_{Z_n} > 0$. Then the result follows.

The reasoning for the third condition can be carried out in the same way as for the second condition.

Thus to compute Z, we need only to apply ref to Z_0 at most $|Z_0|$ times. Note that the computation of ref can be carried out in polynomial time since the optimal value of a linear programming problem can be computed in polynomial time. Hence Z is polynomial-time computable. Then it follows directly that whether a given $d \in \mathrm{FP}$ equals $\boldsymbol{d}_{\mathrm{p}}$ is decidable in polynomial time.

By Theorem 8, we can decide if a given element in FP is $\boldsymbol{d}_{\mathrm{p}}$ in polynomial time. However, it does not tell us how to obtain an element in FP. Below we show that we can guess such an element using only polynomial random-bits. This is also the final step to prove that $\mathcal{L} \in \mathsf{NP} \cap \mathsf{coNP}$. (see the beginning of the section for the definition of $\mathcal{L}$)

Theorem 9. *$\mathcal{L} \in \mathsf{NP} \cap \mathsf{coNP}$.*

Proof. We only show that $\mathcal{L}^{\mathrm{c}} \in \mathsf{NP}$, since proving $\mathcal{L} \in \mathsf{NP}$ is similar. W.l.o.g we assume that $\mathcal{L}^{\mathrm{c}} := \{\langle\langle S, Act, \Omega\rangle, \langle s_{\mathrm{in}}, t_{\mathrm{in}}\rangle, \epsilon\rangle \mid \boldsymbol{d}_{\mathrm{p}}(s_{\mathrm{in}}, t_{\mathrm{in}}) > \epsilon\}$. Let (S, Act, Ω), $(s_{\mathrm{in}}, t_{\mathrm{in}})$ and ϵ be the input. Our strategy to obtain a NP algorithm for $\mathcal{L}^{\mathrm{c}}$ is as follows:

1. We guess a $d \in \mathrm{FP}$ by using Theorem 2 and Theorem 1.
2. We check whether $d = \boldsymbol{d}_{\mathrm{p}}$ by Theorem 8; if $d = \boldsymbol{d}_{\mathrm{p}}$ then we check whether $d(s,t) > \epsilon$, otherwise we fail.

Below we show that how we can guess a $d \in \mathrm{FP}$ using only polynomial bits. The guessing procedure is illustrated step by step as follows.

1. For all $s \xrightarrow{a} \mu$ and $t \in S$ such that $\mathrm{Act}(s) = \mathrm{Act}(t) \neq \emptyset$, we guess a $\nu[s \xrightarrow{a} \mu, t]$ such that $t \xrightarrow{a} \nu[s \xrightarrow{a} \mu, t]$. Then we guess $|S|^2$ constraints (informally "rows") specified in the LP $\Delta_{\mathfrak{d}}^{\mu,\nu[s \xrightarrow{a} \mu,t]}$ (cf. Tab. 1). Note that the parameter $\mathfrak{d}$ is irrelevant here. We denote the guessed collection of constraints (w.r.t $s \xrightarrow{a} \mu$ and t) as $\{h_i^T y \ \mathrm{op}_i \ b^i\}_{i \in I[s \xrightarrow{a} \mu, t]}$, where
   - $y = \{y_{s',t'}\}_{s',t' \in S}$ represents the variable vector;
   - $I[s \xrightarrow{a} \mu, t]$ is the set of indexes of the guessed rows;
   - $h_i^T y \ \mathrm{op}_i \ b^i$ is the constraint at the i-th row, for which $\mathrm{op}_i \in \{\leq, \geq, =\}$ is the comparison operator at the i-th row.

   We check in polynomial time whether for all $s \xrightarrow{a} \mu$ and $t \in S$, the linear equation system $\{h_i^T y = b^i\}_{i \in I[s \xrightarrow{a} \mu, t]}$ has a unique solution (through, e.g., Gaussian Elimination). We fail if the checking result is negative; Otherwise we denote the unique solution to be $y[s \xrightarrow{a} \mu, t]$ (a vector over $S \times S$), and proceed to the next step.
2. For all $t \xrightarrow{a} \nu$ and $s \in S$ such that $\mathrm{Act}(s) = \mathrm{Act}(t) \neq \emptyset$, we perform similar guessings illustrated in Step 1. We guess a $\mu[s, t \xrightarrow{a} \nu]$, and further $|S|^2$ constraints specified in the LP $\Delta_{\mathfrak{d}}^{\mu[s,t \xrightarrow{a} \nu],\nu}$. We denote the guessed constraints as $\{h_i^T y \ \mathrm{op}_i \ b^i\}_{i \in I[s,t \xrightarrow{a} \nu]}$. Then we check whether for all $t \xrightarrow{a} \nu$ and $s \in S$, the linear equation system $\{h_i^T y = b^i\}_{i \in I[s,t \xrightarrow{a} \nu]}$ has a unique solution. We fail if the checking result is negative; Otherwise we denote the unique solution as $y[s, t \xrightarrow{a} \nu]$, and proceed to the next step.

3. We compute a distance function $d \in \mathcal{M}_{\rm d}(S)$ defined by: $d(s,t) = \mathrm{OpV}(\Gamma_{s,t})$ for all $s,t \in S$, where the LP $\Gamma_{s,t}$ (with variables $\{z_{s',t'}\}_{s',t' \in S}$) is specified by:

   minimize $z_{s,t}$
   subject to:
   (i) $z_{s',s'} = 0$, $z_{s',t'} = z_{t',s'}$ and $z_{r',t'} \le z_{r',s'} + z_{s',t'}$ for all $r', s', t' \in S$
   (ii) $0 \le z_{s',t'} \le 1$ for all $s', t' \in S$
   (iii) $z_{s',t'} \ge \sum_{\mathfrak{s},\mathfrak{t} \in S} (y[s' \xrightarrow{a} \mu, t'])_{\mathfrak{s},\mathfrak{t}} \cdot z_{\mathfrak{s},\mathfrak{t}}$ for all $t' \in S$ and all $s' \xrightarrow{a} \mu$ such that $\mathrm{Act}(s') = \mathrm{Act}(t') \neq \emptyset$
   (iv) $z_{s',t'} \ge \sum_{\mathfrak{s},\mathfrak{t} \in S} (y[s', t' \xrightarrow{a} \nu])_{\mathfrak{s},\mathfrak{t}} \cdot z_{\mathfrak{s},\mathfrak{t}}$ for all $s' \in S$ and all $t' \xrightarrow{a} \nu$ such that $\mathrm{Act}(s') = \mathrm{Act}(t') \neq \emptyset$
   (v) $z_{s',t'} = 0$ for all $s', t' \in S$ such that $\mathrm{Act}(s') = \mathrm{Act}(t') = \emptyset$.
   (vi) $z_{s',t'} = 1$ for all $s', t' \in S$ such that $\mathrm{Act}(s') \neq \mathrm{Act}(t')$.

   If there is $\Gamma_{s,t}$ that has no feasible solution (which can also be decided in polynomial time), we fail; Otherwise we proceed to the next step.
4. We check whether $d \in \mathcal{M}_{\rm p}(S)$ and $d = \mathcal{F}_{\rm p}(d)$, which can be done in polynomial time. We fail if the checking is unsuccessful; otherwise we have guessed a $d \in \mathrm{FP}$.

Then (as is previously mentioned), the final step is to check whether $d = \boldsymbol{d}_{\rm p}$: if $d = \boldsymbol{d}_{\rm p}$ then we return the comparison result of $d(s,t) > \epsilon$; otherwise we fail. It is clear that if we can guess a $d \in \mathrm{FP}$ and check successfully that $d = \boldsymbol{d}_{\rm p}$, then the returned result is surely correct. Below we prove that $\boldsymbol{d}_{\rm p}$ can be guessed through the guessing procedure.

Consider Step 1. For any $s \xrightarrow{a} \mu$ and $t \in S$ such that $\mathrm{Act}(s) = \mathrm{Act}(t) \neq \emptyset$, we choose $\nu[s \xrightarrow{a} \mu, t]$ such that $\Delta(\boldsymbol{d}_{\rm p})(\mu, \nu[s \xrightarrow{a} \mu, t]) = \inf\{\Delta(\boldsymbol{d}_{\rm p})(\mu,\nu) \mid t \xrightarrow{a} \nu\}$. By Theorem 2, we can choose $y[s \xrightarrow{a} \mu, t]$ to be a vertex of $\Delta_{\mathfrak{d}}^{\mu,\nu[s \xrightarrow{a} \mu, t]}$ that reaches $\mathrm{OpV}(\Delta_{\mathfrak{d}}^{\mu,\nu[s \xrightarrow{a} \mu,t]})$. Then by Theorem 1, we choose constraints $\{h_i^T y \mathrm{~op}_i~ b^i\}_{i \in I[s \xrightarrow{a} \mu, t]}$ such that $\{h_i^T\}_{i \in I[s \xrightarrow{a} \mu, t]}$ are exactly the row vectors of the matrix $A_{y[s \xrightarrow{a} \mu, t]}$ (cf. Theorem 1). Here by Remark 1, we can abuse Theorem 1 on constraints like "$h^T y \ge e$" and "$h^T y = e$". Then surely the unique solution of $\{h_i^T y = b^i\}_{i \in I[s \xrightarrow{a} \mu, t]}$ is $y[s \xrightarrow{a} \mu, t]$. Similarly, we choose $y[s, t \xrightarrow{a} \nu]$ for all $s \in S$ and $t \xrightarrow{a} \nu$ such that $\mathrm{Act}(s) = \mathrm{Act}(t) \neq \emptyset$ in Step 2.

Then consider Step 3. We show that the $d \in \mathcal{M}_{\rm d}(S)$ defined in Step 3 is equal to $\boldsymbol{d}_{\rm p}$, provided that $y[s' \xrightarrow{a} \mu, t']$ and $y[s', t' \xrightarrow{a} \nu]$ are chosen as above.

1. $\boldsymbol{d}_{\rm p} \preceq_{\rm d} d$: By the definition of pseudometrics and Remark 2, the vector $\{z^*_{s',t'}\}_{s',t' \in S}$ is a feasible solution of $\Gamma_{s,t}$ for all $s,t \in S$, where $\{z^*_{s',t'}\}_{s',t' \in S}$ is given by: $z^*_{s',t'} = \boldsymbol{d}_{\rm p}(s',t')$ for all $s',t' \in S$. Thus immediately we have $\boldsymbol{d}_{\rm p} \preceq_{\rm d} d$.
2. $d \preceq_{\rm d} \boldsymbol{d}_{\rm p}$: Fix arbitrary $(s,t) \in S \times S$ and let $\{\mathbf{z}_{s',t'}\}_{s',t' \in S}$ be an optimal solution of $\Gamma_{s,t}$. Define $d' \in \mathcal{M}_{\rm d}(S)$ by: $d'(s',t') = \mathbf{z}_{s',t'}$ for arbitrary $s',t' \in S$. Then for any $t' \in S$ and $s' \xrightarrow{a} \mu$ such that $\mathrm{Act}(s') = \mathrm{Act}(t') \neq \emptyset$,

$$d'(s',t') \ge \sum_{\mathfrak{s},\mathfrak{t} \in S} (y[s' \xrightarrow{a} \mu, t'])_{\mathfrak{s},\mathfrak{t}} \cdot d'(\mathfrak{s},\mathfrak{t}) \ge \Delta(d')(\mu, \nu[s' \xrightarrow{a} \mu, t'])$$

   Hence $d'(s',t') \ge \inf\{\Delta(d')(\mu,\nu) \mid t' \xrightarrow{a} \nu\}$. In a similar way, we can prove that $d'(s',t') \ge \inf\{\Delta(d')(\mu,\nu) \mid s' \xrightarrow{a} \mu\}$ for all $s' \in S$ and $t' \xrightarrow{a} \nu$ such that $\mathrm{Act}(s') = \mathrm{Act}(t') \neq \emptyset$. Thus $d' \preceq_{\mathrm{d}} \mathcal{F}_{\mathrm{d}}(d')$ and hence $d' \preceq_{\mathrm{d}} \boldsymbol{d}_{\mathrm{p}}$. Then $d(s,t) \geq \boldsymbol{d}_{\mathrm{p}}(s,t)$ since $d(s,t) = d'(s,t)$.

In conclusion, $\boldsymbol{d}_{\mathrm{p}}$ can be obtained on certain random bits in the guessing procedure.

5 Conclusion and Future Work

We have shown that the problem whether the distance between two given states of a finite simple probabilistic automata under a non-future-discounting pseudometric derived from a real-valued logic lies in $\mathsf{NP} \cap \mathsf{coNP}$, which significantly improves previous PSPACE upperbound [vBSW07,CHL09]. We prove this by establishing the original notion of "self-fulfilling" sets, and then exploring the relationship between our pseudometric and self-fulfilling sets. Our future work is to investigate if the problem further lies in PTIME, and whether there is an efficient error-tolerant algorithm for this problem. It is also possible to extend the result of this paper to game metrics [dAMRS07].

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